The Open University

MU123

Discovering mathematics

HANDBOOK

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Handbook

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The Open University, Walton Hall, Milton Keynes, MK7 6AA.

First published 2010. Second edition 2014.

Edited, designed and typeset by The Open University, using the Open University TEX System.

Printed in Great Britain by Henry Ling Limited, at the Dorset Press, Dorchester DT1 1HD

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Contents

1 Abbreviations

Some of the abbreviations used in MU123 are listed below. In the margin next to each entry is a reference to the unit and page of MU123 where the notation is first used.

C8 31	AAA	angle-angle-angle condition for similar (not congruent) triangles
C8 31	AAS	angle-angle-side condition for congruent triangles
D13 134	APR	annual percentage rate
C8 31	ASA	angle-side-angle condition for congruent triangles
C8 31	ASS	angle-side-side (not a condition for similar or congruent triangles)
A1 13	BIDMAS	brackets, indices, divisions and multiplications, additions and subtractions
D14 199	CAST	cosine, all, sine, tangent (mnemonic for which trigonometric ratios are positive in which quadrants, starting from the bottom right and going round anticlockwise)
A1 19	d.p.	decimal place(s)
C9 82	FOIL	first, outer, inner and last (order for multiplying out brackets)
A3 124	HCF	highest common factor
A4 202	IQR	interquartile range
A3 121	LCM	lowest common multiple
B5 35	LHS	left-hand side of an equation
A4 180	PCAI	pose question, collect relevant data, analyse the data, interpret the results (the four stages of a statistical investigation)
B5 35	RHS	right-hand side of an equation
C8 31	SAS	side-angle-side condition for congruent triangles
A4 205	SD	standard deviation
A1 20	s.f.	significant figure(s)
A1 15	SI	standard metric system (Système Internationale d'Unités)
D12 61	SOH CAH TOA	sine is opposite over hypotenuse; cosine is adjacent over hypotenuse; tangent is opposite over adjacent (mnemonic for trigonometric ratios of side lengths in a right-angled triangle)
C8 31	SSS	side-side-side condition for congruent triangles

2 Notation

Some of the notation used in MU123 is listed below. In the margin next to each entry is a reference to the unit and page of MU123 where the notation is first used.

A1 20	$\approx$	is approximately equal to
A1 24	$\dots$	ellipsis symbol, indicating that something has been omitted
A1 37	$\%$	percent
A2 98 and **C8 9**	$^\circ$	a degree: an indication of a measurement on the Celsius or Fahrenheit temperature scales, or $1/360$ th of a full turn.

$<$	less than	**A2 103**
$\leq$ or $\leqslant$	less than or equal to	**A2 104**
$>$	greater than	**A2 103**
$\geq$ or $\geqslant$	greater than or equal to	**A2 104**
a^n	a to the power n	**A3 131**
$0.1\dot{2}9\dot{6}$ or $0.1\overline{296}$	the recurring decimal $0.1296\,296\,296\,296\,296\ldots$	**A3 137**
a^{-n}	$1/a^n$, where $a \neq 0$	**A3 144**
$\pm$	plus or minus	**A3 151**
$\sqrt{a}$	the non-negative square root of a, where $a \geq 0$	**A3 151**
$\sqrt[n]{a}$	the non-negative nth root of a, where $a \geq 0$	**A3 152**
$a^{m/n}$	$(\sqrt[n]{a})^m$, where $a \geq 0$	**A3 157**
$m : n$	the ratio m to n	**A3 159**
Q1	the lower quartile	**A4 202**
Q3	the upper quartile	**A4 202**
(x, y)	the coordinates of a point (also used to label the point)	**B6 63**
$P(x, y)$	the point P with coordinates (x, y)	**B6 63**
x-, y-	prefixes placed before words like axis, coordinate and intercept. Here x is the standard variable associated with the horizontal axis, and y is the standard variable associated with the vertical axis, but other variables are often used in applications; the x or y in the prefix is then replaced by the appropriate variable.	**B6 63**
$y \propto x$	y is directly proportional to x	**B6 88**
r	the correlation coefficient	**B6 107**
$\neq$	not equal to	**B7 127**
AB	the line segment between points A and B, or its length	**C8 9**
$\angle ABC$	the angle formed from line segments AB and BC, or its size	**C8 9**
$A\hat{B}C$	the angle formed from line segments AB and BC, or its size	**C8 9**
∟	the square symbol indicating a right angle	**C8 9**
$\gg$	one or more arrowheads (on two or more lines) indicating parallel lines	**C8 12**
∢	one or more arcs (on two or more angles) indicating equal angles	**C8 14**
$\triangle ABC$	the triangle with vertices A, B and C	**C8 17**
$\|\|$	one or more strokes (on two or more line segments) indicating line segments of equal length	**C8 19**
$\cong$	is congruent to	**C8 30**
π	the ratio of the circumference of a circle to its diameter (≈ 3.142)	**A2 95** and **C8 55**
g	the acceleration due to gravity ($\approx 9.8\,\mathrm{m/s^2}$)	**C10 129**
$\sin\theta$	the sine of the angle θ	**D12 61**
$\cos\theta$	the cosine of the angle θ	**D12 61**
$\tan\theta$	the tangent of the angle θ	**D12 61**

D12 69	$\sin^{-1}(x)$ or $\arcsin(x)$	the inverse sine of x
D12 69	$\cos^{-1}(x)$ or $\arccos(x)$	the inverse cosine of x
D12 69	$\tan^{-1}(x)$ or $\arctan(x)$	the inverse tangent of x
D13 146	e	Euler's number (≈ 2.718)
D13 147	$\exp x$	the value e raised to the power x, that is, e^x
D13 152, 155	$\log x$	the logarithm of x to an unspecified base, or the logarithm to base 10 of x. (In some disciplines, $\log x$ can mean $\ln x$.)
D13 154	$\log_b x$	the logarithm to base b of x
D13 155	$\ln x$	the logarithm to base e of x
D14 224	$\|x\|$	the magnitude of x

3 Glossary

This glossary presents some of the key terms used in MU123. In the margin next to each entry is a reference to the unit and page of the book where the term is first used or defined. If appropriate this is followed by an italicised reference in brackets to a page in this handbook where further details can be found. Within definitions, cross-references to related glossary items are italicised.

A1 39 **absolute comparison** See *relative comparison.*

B6 74 **absolute value** See *magnitude (of a number).*

A2 78 **accuracy (of an answer)** How close the answer is to the correct value.

A4 213 **accuracy (of a set of measurements)** How close the average of a set of repeated measurements is to the true value.

C8 10 **acute angle** An *angle* greater than 0° and less than 90°.

D12 60 **adjacent (angle and side)** In a triangle, an angle and side are adjacent if the side is one of the two line segments that form the angle.

C8 23 **adjacent (sides)** Two sides of a shape that meet at a vertex of the shape.

B6 67 *(35)* **against** On the *graph* of a *formula*, the *dependent variable* is usually put on the *vertical axis* and is said to be plotted against the *independent variable*, which is put on the *horizontal axis.*

B5 6 **algebra** A branch of mathematics in which letters are used to represent numbers.

B5 11 *(34, 36)* **algebraic expression** See *expression.*

B5 30 **algebraic fraction** An algebraic *expression* that has the form of a *fraction.*

A2 76 **algorithm** A set of instructions to solve a problem step by step.

C8 14 *(38)* **alternate angles** Two of the *angles* formed when a *line* l crosses a pair of *parallel* lines are said to be alternate if they lie between the parallel lines on opposite sides of l and have different vertices. Alternate angles on parallel lines are equal.

amplitude (of a sinusoidal curve) Half the difference between the maximum and minimum values of the curve. **D14 226** *(50)*

angle There are three related meanings of the term angle: (1) a measure of rotation, often expressed in *degrees* or *radians* (see also *sign of an angle*); (2) a configuration consisting of two *line segments* emerging from a *vertex*; (3) the size of a rotation (usually the smallest) that makes one line segment of such a configuration lie in the same direction as the other. **C8 9** *(38)*

angle of elevation The angle between the horizontal and the line of sight to an object. **D12 59**

angle of inclination The angle (greater than or equal to 0° and less than 180°) that a line makes with the positive x-axis, when the line is drawn on a pair of axes with equal scales. **D14 201** *(50)*

annual percentage rate (APR) The percentage of a loan, or of savings, that is to be paid as interest each year. **D13 134**

antilogarithm The antilogarithm (to *base* b) of a number x is the number whose *logarithm* (to base b) is x. **D13 149**

apex (of a cone) See *cone*. **C8 59**

apex angle The *angle* formed by the equal sides of an *isosceles triangle*. **C8 19**

arc (of a circle) An unbroken section of the *circumference* of a *circle*. **C8 54**

arccosine Another name for *inverse cosine*. **D12 69**

arcsine Another name for *inverse sine*. **D12 69**

arctangent Another name for *inverse tangent*. **D12 69**

area (of a shape) The amount of surface that a shape occupies. **C8 48** *(39)*

arithmetic mean The arithmetic mean (or just mean) of a set of numbers is the sum of the numbers divided by however many numbers there are in the set. **A4 196** *(33)*

arithmetic progression Another name for *arithmetic sequence*. **C9 75** *(40)*

arithmetic sequence A *sequence* in which the *difference* between successive *terms* is constant. For example, 2, 2.5, 3, 3.5, 4, ... **C9 75** *(40)*

aspect ratio The aspect ratio of a rectangle is the ratio of its longer side to its shorter side. **A3 163**

asymptote A *line* that a *graph* approaches but never reaches. An asymptote is often indicated on a graph by a dashed line. **D12 96**

average speed The average *speed* is calculated by dividing the distance travelled by the time taken. **A2 72** *(30)*

axiom A truth that is taken as self-evident. **C8 8**

axis of symmetry See *line of symmetry*. **C10 138** *(43)*

ballistics The science of *projectiles*. **C10 132**

bar chart A diagram used to represent discrete numerical or *categorical data*. Each numerical or categorical item is represented by a *rectangle*, called a bar or column, whose length is *proportional* to the numerical value associated with that item (this could be its frequency of occurrence, or something else). The bars are of equal thickness and they have bases along either a *horizontal* or a *vertical axis*. There are equal gaps between the bars. **D11 22**

base (number) See *index form*. **A3 131** *(32)*

C8 49 *(39)* **base (of a shape)** The side (or face) of a shape at right angles to which the *perpendicular height* is to be measured.

C8 19 **base angles** The two equal *angles* of an *isosceles triangle.*

A1 15 **base units** Units of measurement from which all others are derived.

B6 105 **best fit line** See *regression line.*

A1 13 *(28)* **BIDMAS** An acronym that acts as a reminder of the order in which to carry out mathematical operations.

A4 185 **binary data** *Data* that can take only two values, often 1 and 0, and are widely used to represent categories such as yes/no, pass/fail or true/false.

C8 36 **bisect** To cut into two equal parts.

D11 10 *(45)* **boxplot** A diagram used to represent five key summary values of a *dataset*, namely minimum, *lower quartile*, *median*, *upper quartile* and maximum. A box is drawn between the lower and upper quartile to indicate the *interquartile range*, while two line segments (known as whiskers) are drawn between the box and the minimum and maximum values of the dataset to indicate its full *range.*

C9 103 **cancelling (an algebraic fraction)** The process of cancelling any common factors of the numerator and denominator.

A1 34 *(29)* **cancelling (a numerical fraction)** The process of dividing the top and bottom of a *fraction* by a whole number (larger than 1) to obtain an *equivalent fraction* with a smaller *numerator* and *denominator.*

B5 18 *(35)* **cancelling out terms** Two or more *like terms* of an *expression* cancel each other out if their *coefficients* add up to zero.

C8 60 **capacity** The amount of liquid that an object can contain.

B6 63 **Cartesian coordinate system** A way of specifying the position of a point using *coordinates.*

D11 22 **categorical data** *Data* that have been classified according to a set of categories. For example, the following categories might be used in connection with housing: Detached houses, Semi-detached houses, Terraced houses, Purpose-built flats, Converted flats and Other.

C8 54 **centre (of a circle or sphere)** See *circle* and *sphere.*

D12 104 **centre (of a circular arc)** The *centre* of the *circle* on whose *circumference* the *arc* lies.

C8 27 **centre of rotation** The *point* about which something is rotated.

D14 181 **chance** Another name for *probability.*

C8 54 **chord** A *line segment* starting and ending on the *circumference* of a *circle.*

C8 54 *(39)* **circle** A *plane shape* whose boundary consists of all *points* that are a fixed distance from a fixed point called the centre of the circle. The word circle is also used to refer to the boundary of such a shape.

D12 104 **circular arc** Another name for *arc (of a circle).*

C8 54 *(39)* **circumference (of a circle)** The boundary of a *circle*, or the length of the boundary.

C8 56 **circumscribe** To construct (a shape) around another shape so that it touches but does not cross that other shape.

clearing a fraction The process of removing a *fraction* (numerical or algebraic) from an *equation* by multiplying both sides of the equation by a suitable number or expression. (The number must be non-zero, and the expression must be non-zero for all values of the variables under consideration.) **B5 43** and **C9 110** *(35, 42)*

coefficient When a *term* in an *expression* consists of a number (including any signs) multiplied by a combination of letters, the number is called the coefficient of the term. **B5 15**

coefficient (of a quadratic expression) The coefficients of the *quadratic expression* $ax^2 + bx + c$ are the constants a, b and c. **C9 87**

collecting like terms The process of combining *like terms* of an expression into a single *term*. **B5 16** *(35)*

common denominator A common denominator of two or more *fractions* is a *common multiple* of their *denominators*. **A3 138** and **C9 105** *(31, 42)*

common difference The *difference* between successive *terms* in an *arithmetic sequence*. **C9 75** *(40)*

common factor (of integers) A common factor of two or more *integers* is an integer that is a *factor* of them all. **A3 124** *(31)*

common factor (of terms) A common factor of two or more *terms* is an *expression* that is a *factor* of them all. **B7 134** *(36)*

common logarithm The common logarithm of a number x is the power to which 10 has to be raised to obtain x (that is, the logarithm of x to base 10). This is sometimes written with the subscript omitted, as $\log x$. **D13 150**

common multiple (of expressions) A common multiple of two or more *expressions* is an expression that is a *multiple* of them all.

common multiple (of integers) A common multiple of two or more *integers* is an integer that is a *multiple* of them all. **A3 121** *(31)*

common side A *line segment* that is a side of two shapes. **C8 36**

comparative bar chart A *bar chart* in which there are two or more bars for each data item. For example, two bars could be associated with each year, one to represent the number of mobile phones, and the other the number of land lines. **D14 186**

completed-square form Every *quadratic expression* $ax^2 + bx + c$ can be *rearranged* into the form $a(x + \text{a number})^2 + \text{a number}$. This is known as the completed-square form of the quadratic expression. **C10 162** *(44)*

completing the square The process of *rearranging* a *quadratic expression* into its *completed-square form*. **C10 162** *(44)*

composite number An *integer* greater than 1 that is not a prime. **A3 126**

compound interest Interest that is a *percentage* of both the initial amount of an investment and all the interest accumulated so far. **D13 122**

compound unit A unit of measurement that involves more than one of the *base units*, such as m/s or m^3. **A2 72**

cone A *three-dimensional shape* with a circular *base*, whose cross-sections (parallel to the base) are *circles* that decrease in *radius* uniformly to a *point*, known as the apex of the cone. The centres of the circular cross-sections form a straight line perpendicular to the base. **C8 59** *(40)*

congruent Geometric *figures* with the same size and shape (possibly flipped) are said to be congruent. **C8 28** *(39)*

A1 46 **conjecture** An informed guess about what might be true, often obtained by considering some special cases.

B6 88 **constant** A constant in an *equation* or *expression* is a quantity that does not change when the values of the *variables* change. Sometimes 'constant' is used as a short form of *constant term*.

B6 88 **constant of proportionality** See *direct proportion*.

B5 15 **constant term** A *term*, in an *expression*, that is just a number.

C8 17 **construction** An addition to a geometric *figure*, used to help prove a *result* about the original figure.

C8 17 **construction line** A *line* used as (part of) a *construction*.

A4 186 **continuous data** *Data* that can take all the 'in-between' values on a number scale.

D13 125, 137 **continuous exponential change** See *exponential change*.

C8 16 **converse** The reverse of a mathematical statement. The converse of the statement 'If A is true, then B is true' is the statement 'If B is true, then A is true.'

A2 84 **conversion graph** A *graph* used to convert from one unit to another unit, for instance m/s to km/h.

A2 85 and B6 63 **coordinates** A pair of numbers used to represent a point. The first number specifies the position of the point along the *horizontal axis* from 0, and the second number specifies its position along the *vertical axis* from 0. These numbers are known as the horizontal coordinate and the vertical coordinate, respectively.

B6 108 **correlation** See *positive correlation*, *negative correlation*, *perfect correlation*.

B6 107 **correlation coefficient** The correlation coefficient of a set of *paired data* measures how closely the *regression line* fits the *data points*. A value close to $+1$ indicates a strong *positive correlation*, whereas a value close to -1 indicates a strong *negative correlation*; the closer the value is to 0, the weaker is the correlation. A correlation coefficient is sometimes denoted by r.

C8 14 *(38)* **corresponding angles (on parallel lines)** Two of the *angles* formed when a *line* l crosses a pair of *parallel* lines are said to be corresponding if they have different vertices and lie on the same side of l with just one of them between the parallel lines. Corresponding angles on parallel lines are equal.

C8 30 *(39)* **corresponding angles or vertices (of congruent triangles)** An *angle* (or *vertex*) in one triangle is said to correspond to an angle (or vertex) in a *congruent* triangle if the two angles (or two vertices) can be made to coincide by superimposing the two triangles (flipping one if necessary).

C8 34 **corresponding sides (of congruent or similar triangles)** If two triangles have the same three *angles*, then a side in one triangle is said to correspond to a side in the other triangle if they are opposite equal angles.

D12 61, 90 *(45)* **cosine** The cosine of an *angle* θ, written $\cos\theta$, is the x-coordinate of the *point* obtained by rotating the point $(1, 0)$ about the *origin* through the angle θ. For an acute angle θ in a *right-angled triangle*, $\cos\theta$ is equal to the length of the side *adjacent* to θ divided by the length of the *hypotenuse*.

cosine curve The graph of the *cosine* function. **D12 95**

Cosine Rule A rule for finding the length of one side of a *triangle* given the lengths of the other two sides and an *angle*, or for finding an angle given the lengths of the three sides. **D12 80** *(45)*

critical region In certain types of statistical test, the *null hypothesis* is rejected if a suitably-chosen measure lies in a critical region. The critical region is chosen so that if the hypothesis were true, then there would be only a certain chance, often chosen to be 5%, of the measure lying in the critical region. The value at which a critical region starts is known as a critical value. **D11 46**

critical value See *critical region.* **D11 46**

cross multiplication A method of *clearing the fractions* in an *equation* that consists of a fraction on each side. The *numerator* on each side is multiplied by the *denominator* on the other side and the results are equated. **D14 193** *(42)*

cube A *prism* with square cross-section and square sides. **C8 59**

cube (of a number) The cube of a number is the number raised to the *power* 3. **A3 131** *(32)*

cube root A cube root of a number is a number whose *cube* is the original number. **A3 151** *(32)*

cuboid A *prism* with rectangular cross-section and rectangular sides. **C8 59** *(40)*

cycle (of a sinusoidal curve) Another name for *oscillation.* **D14 222**

cylinder A *three-dimensional shape* formed by filling in the space between two parallel congruent *circles.* The straight line joining the centres of the circles is perpendicular to the circles. **C8 59** *(40)*

data Data are facts or statistics. **A2 77**

data point A point plotted on a *scatterplot.* Also, another name for a data value. **B6 68**

dataset A dataset is a collection of *data*, usually in tabular form. **A4 183**

decagon A *polygon* with ten sides. **C8 23**

decimal places (d.p.) The positions of *digits* to the right of the decimal point. Also used to indicate the *precision* of an answer, for example 'to three decimal places (to 3 d.p.)' means 'to three-digit precision after the decimal point'. **A1 19** *(28)*

degree A degree (indicated by °) is 1/360 th of a full turn. It also means an increment on the Celsius or Fahrenheit temperature scales. **A2 98** and **C8 9**

denominator The bottom number or expression in a numerical or algebraic fraction. **A1 33** and **B5 30** *(29, 32)*

dependent variable In a practical formula, the subject is often referred to as the dependent variable and the other variable as the independent variable. **B6 67** *(35)*

depreciation The decline in the value of an item. **D13 135**

diameter A *chord* that passes through the centre of a *circle*, or the length of such a chord. **C8 54**

difference A difference between two numbers is the result of subtracting one from the other, usually the smaller from the larger. **A1 14** *(28, 31)*

A1 18 *(28)* **digit** A (decimal) digit is one of the symbols $0, 1, \ldots, 9$.

C8 54 **dimensionless quantity** A quantity that has no units associated with it – that is, a pure number.

B6 87, 88 **direct proportion** Two quantities x and y are (directly) proportional to each other if they are related by an *equation* of the form $y = kx$, where k is a non-zero *constant* known as the constant of proportionality.

A4 185 **discrete data** *Data* that can take one of a particular set of separated values (such as a set of *integers* or the set of shoe sizes).

D13 125, 134 *(47)* **discrete exponential change** See *exponential change.*

C10 157 *(44)* **discriminant** The value $b^2 - 4ac$ is called the discriminant of the *quadratic expression* $ax^2 + bx + c$.

A3 122 *(31)* **divisible** Capable of being divided without a remainder.

A3 122 *(31)* **divisor (of an integer)** See *factor (of an integer).*

D11 9 **dotplot** A pictorial representation of a *dataset* using columns of dots above a *horizontal axis.* Each data value is represented by the position of one of the dots along the axis.

A2 106 *(30)* **double inequality** A combination of two *inequalities* in the same variable, such as $-2 \leq a < 5$ which means $-2 \leq a$ and $a < 5$.

D13 166 *(48)* **doubling time** The time it takes for a quantity that grows exponentially to double in size.

D12 76 **dropping a perpendicular** The process of drawing a *line* through a *point* in a direction at right angles to a given line is known as dropping a perpendicular from the point to the given line.

B7 146 **eliminating (an unknown)** The process of combining two or more *equations* to obtain a new equation with fewer *unknowns.*

B5 12 **equation** Two *expressions* with an equals sign between them.

B5 37 *(35)* **equation in one unknown** An *equation* in which a single *unknown* appears one or more times.

C8 19 *(38)* **equilateral triangle** A triangle that has all its sides the same length.

B5 42 **equivalent (equations)** Two *equations* are said to be equivalent, or different forms of the same equation, if one can be *rearranged* to give the other.

B5 12 *(34)* **equivalent (expressions)** Two *expressions* are said to be equivalent, or different forms of the same expression, if they yield a common value for each substitution of the letters.

A1 33 *(29)* **equivalent (fractions)** When you multiply or divide the *numerator* and *denominator* of a *fraction* by the same non-zero whole number (or non-zero expression, in the case of an algebraic fraction) you obtain an equivalent fraction.

A3 160 *(32)* **equivalent (ratios)** When you multiply or divide each number in a *ratio* by the same non-zero number you obtain an equivalent ratio.

D13 146 **Euler's number ($e \approx 2.718$)** The value b for which the graph of $y = b^x$ has a gradient of 1 at $(0, 1)$.

B5 11 **evaluating (an expression)** The process of substituting numbers for the letters in an *expression* and calculating its value.

A1 44 **even (integer)** An even *integer* is one that is divisible by 2.

expanding (an algebraic fraction) The process of dividing each *term* in the *numerator* of the *algebraic fraction* by the *denominator*. **B5 31** *(34)*

expanding the brackets See *multiplying out the brackets*. **B5 25** *(34)*

exponent See *index form*. **A3 131** *(32)*

exponential change A *variable* y is said to change exponentially with respect to a variable x if the relationship between x and y is given by an equation of the form $y = ab^x$, where a and b are positive *constants*, with b not equal to 1. If $b > 1$, then y is said to grow exponentially. If $0 < b < 1$ then y is said to decay exponentially. If the change happens in steps (x takes values from a range of equally-spaced numbers, such as the non-negative *integers*), then it is discrete exponential change (also called geometric change). If the change happens continuously (x takes values from an *interval* of real numbers, such as the non-negative *real numbers*), then it is continuous exponential change. **D13 119, 120, 125** *(47)*

exponential (growth/decay) curve The *graph* of an *equation* of the form $y = ab^x$, where a and b are positive *constants*, with b not equal to 1. The curve is known as an exponential growth curve if $b > 1$, and as an exponential decay curve if $0 < b < 1$. **D13 126** *(48)*

exponential decay See *exponential change*. **D13 120, 125**

exponential equation An *equation* in which the *unknown* is in an *exponent*, such as $2^{x+1} = 5$. **D13 148** *(48)*

exponential function A *function* whose rule is of the form $y = b^x$ for some positive *constant* b that is not equal to 1. See also *the exponential function*. **D13 138** *(48)*

exponential growth See *exponential change*. **D13 119, 125**

exponential model A *model* based on a *formula* of the form $y = ab^x$, where a and b are positive *constants* with b not equal to 1. **D13 126**

exponential regression The process of fitting an *exponential curve* as closely as possible to a given set of paired data. **D13 129**

expression An algebraic expression, or just expression, is a collection of letters, numbers and/or mathematical symbols, arranged in such a way that if numbers are substituted for the letters, then you can work out the value of the expression. An expression does not have to contain letters, but the term 'algebraic expression' is usually used only if it does. **B5 11** *(34, 36)*

exterior angle (of a polygon) An *angle* outside the *polygon* formed by a side and an extended *adjacent* side. **C8 23**

extrapolation The process of using a set of paired data to estimate a new data point that lies to the left of all the data points given by the paired data, or to the right of all of them. **B6 106**

F angles An informal name for *corresponding angles (on parallel lines)*. **C8 14** *(38)*

factor (of an integer) An *integer* that divides a second integer exactly is called a factor, or divisor, of that second integer. (Sometimes, such as when the natural numbers are being considered, the words 'factor' and 'divisor' are used to refer to positive factors only.) **A3 122** *(31)*

factor (of a term) If a *term* can be written in the form something × something (by reordering its letters, factorising its coefficients, and so on), then each 'something' is a factor of the term. **B7 134** *(36)*

A3 122 *(31)*	**factor pair** A pair of *integers* whose product is equal to a given integer is called a factor pair of that integer. (Sometimes, such as when the natural numbers are being considered, the term 'factor pair' is used to refer to pairs of positive factors only.)
A3 126	**factor tree** A tree-like diagram illustrating how a *factorisation* has been carried out.
B7 136 *(36)*	**factorisation (of an expression)** The reverse of *multiplying out the brackets*.
A3 127 *(31)*	**factorisation (of an integer)** The process of writing an *integer* as a product of *factors* that are integers not equal to 1 or -1. (Sometimes, such as when the natural numbers are being considered, the word 'factorisation' is used to refer to products of positive factors only.)
C9 75	**finite sequence** A *sequence* with a finite number of *terms*.
A2 89	**formula** An equation in which one *variable*, called the subject of the formula, appears by itself on the left-hand side of the equation and not at all on the right-hand side, e.g. $y = 3x + 2$. The word 'formula' is sometimes used more loosely, to mean the expression on the right-hand side of such an equation, e.g. '$3x + 2$ is a formula for y', or any equation relating two or more variables, e.g. 'x and y are related by the formula $y - 3x = 2$'.
A1 50	**fractal** A shape that is irregular at all scales, no matter how closely it is viewed. Many fractals can be split into parts, each of which is (at least approximately) a reduced-size copy of the whole. A shape that has this property is said to be self-similar.
B5 30	**fraction (algebraic)** See *algebraic fraction*.
A1 33 *(29)*	**fraction (numerical)** A number that describes the relationship between part of something and the whole. A fraction consists of two *integers*: one, the *denominator*, indicates how many parts of something make up a whole; the second, the *numerator*, indicates how many of these parts the fraction specifies.
C10 129	**free-fall equation** An *equation* relating the distance that an object has fallen to the time that it has taken to fall.
D11 21	**frequency diagram** A diagram that shows the frequencies of particular items, values or groups of values.
B6 86	**function** A rule that takes input values and produces output values.
D14 222 *(50)*	**general cosine function** A *function* whose rule has the form $y = a\cos(b(x - c)) + d$, for some constants a, b, c, d, with a and b non-zero.
D14 222 *(50)*	**general sine function** A *function* whose rule has the form $y = a\sin(b(x - c)) + d$, for some constants a, b, c, d, with a and b non-zero.
D13 125	**geometric change** See *exponential change*.
B6 69, 70, 78 *(35)*	**gradient** The gradient (or slope) of the line through the points (x_1, y_1) and (x_2, y_2) is $\frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$. It is a measure of how steep the line is.
A2 84 and **B6 64–65** *(30, 35)*	**graph** A diagram showing the relationship between two variables. Typically the relationship is illustrated by associating each *variable* with an axis, one horizontal and one vertical. Points are plotted whose horizontal and vertical *coordinates* correspond to related values of the variables. A smooth curve (or straight line if appropriate) is then often drawn through the points to indicate the relationship's more general behaviour.

greater than ($>$) A number is greater than another number if it lies to the right of that number on the *number line*. For example, $-1 > -3$. — **A2 104** *(30)*

greatest common divisor (GCD) Another name for *highest common factor*. — **A3 124** *(31)*

half-life Another term for *halving time*, often used in the context of radioactive decay. — **D13 167**

halving time The time it takes for a quantity that decays exponentially to halve in size. — **D13 167** *(48)*

hemisphere Either of the *solids* obtained by cutting a *sphere* with a *plane* through its centre. — **C8 62**

heptagon A *polygon* with seven sides. — **C8 23**

hexagon A *polygon* with six sides. — **C8 23**

highest common factor (HCF) (of integers) The highest common factor of two or more *integers* is the largest integer that is a *factor* of them all. — **A3 124** *(31)*

highest common factor (of terms) One *common factor* of two or more *terms* is said to be higher than a second common factor if the second is a *factor* of the first and the first is not a factor of the second. (For example, ab is a higher common factor of the terms a^2b and $2abc$ than the common factor a.) A highest common factor of two or more terms is a common factor of the terms such that no other common factor is higher. — **B7 135** *(36)*

histogram A diagram that represents a *dataset* by grouping it into contiguous *intervals* along a *horizontal axis*. Each interval forms the *base* of a *rectangle* whose area (or height if the intervals are of equal width) is *proportional* to the frequency (number of occurrences) of data values in the interval. — **D11 21**

horizontal axis A horizontal line with a scale that is used to specify the horizontal position of a point. — **A2 84** and **B6 64** *(35)*

horizontal coordinate See *coordinates*. — **A2 85** and **B6 64**

horizontal displacement (of a sinusoidal curve) The amount by which the point at $(0, 0)$ on the *sine curve* is displaced to the right when the curve is shifted, stretched and/or compressed to obtain a *sinusoidal curve*. — **D14 226** *(50)*

horizontal intercept See *intercept*. — **B6 83** *(35, 43)*

hypotenuse In a right-angled triangle, the longest side, opposite the *right angle*, is called the hypotenuse. — **C8 43** *(39)*

identity An *equation* that is true for all (appropriate) values of its *variables*. — **B5 12**

improper fraction Another name for *top-heavy fraction*. — **A1 35**

included angle An *angle* between two *adjacent* sides of a shape is called the included angle (of the two sides). — **C8 32**

included side A side of a shape between two *angles* is called the included side (of the two angles). — **C8 32**

independent events Two events are independent if the occurrence (or not) of one event is not influenced by whether the other occurs. — **D11 31**

independent variable See *dependent variable*. — **B6 67**

A3 131 *(32)* — **index** See *index form*.

A3 131 *(32)* — **index form** The result of 'raising a number a to the *power* n' is usually written a^n. This is known as index form or index notation. The number a is called the base number or base and n is called the power, index or exponent.

A3 135 *(32)* — **index laws** Rules that may be used when working with numbers in index form.

A3 131 *(32)* — **index notation** See *index form*.

A2 104 *(30)* — **inequality** Any statement involving one or more of the *inequality signs*.

A2 104 *(30)* — **inequality sign** Any of the four symbols $<$, $\leq$, $>$ and $\geq$.

A1 50 — **infinite** Endless and without limit.

C9 75 — **infinite sequence** A *sequence* with an *infinite* number of *terms*.

C8 56 — **inscribe** To construct (a shape) within another shape so that it touches but does not cross that other shape.

A1 28 — **integer** Any of the numbers $\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$; that is, the negative whole numbers, zero, and the positive whole numbers.

B6 82 *(35, 43, 48)* — **intercept** A value on a *graph* axis scale where the curve (or straight line) meets the axis. An intercept on the *horizontal axis* is known as a horizontal intercept or x-intercept, and an intercept on the *vertical axis* is known as a vertical intercept or y-intercept.

C8 17 — **interior angle (of a polygon)** An *angle* inside the *polygon* formed by two *adjacent* sides.

B6 106 — **interpolation** The process of using a set of paired data to estimate a new data point whose horizontal coordinate lies between the horizontal coordinates of two of the data points given by the paired data.

A4 202 *(33)* — **interquartile range (IQR)** The difference Q3 − Q1 between the *upper quartile* (Q3) and the *lower quartile* (Q1) of a *dataset*.

A2 105 *(30)* — **interval** A section of the *number line* without any gaps.

D12 69 — **inverse cosine** The inverse cosine of a number x, denoted by $\cos^{-1}(x)$ or $\arccos(x)$, is the angle between $0°$ and $180°$ (inclusive) whose *cosine* is x.

D13 169 *(49)* — **inverse functions** Two *functions* whose rules undo each other's effects.

D13 153 — **inverse operations** Two operations that undo each other's effects.

D12 69 — **inverse sine** The inverse sine of a number x, denoted by $\sin^{-1}(x)$ or $\arcsin(x)$, is the angle between $-90°$ and $90°$ (inclusive) whose *sine* is x.

D12 69 — **inverse tangent** The inverse tangent of a number x, denoted by $\tan^{-1}(x)$ or $\arctan(x)$, is the angle between $-90°$ and $90°$ (exclusive) whose *tangent* is x.

A3 150 — **irrational number** A real number that is not a *rational number*, for example $\sqrt{2}$.

C8 19 *(38)* — **isosceles triangle** A triangle with just two equal sides.

C8 24 *(38)* — **kite** A *quadrilateral* with two pairs of *adjacent* equal sides.

A3 121 *(31)* — **least common multiple** Another name for *lowest common multiple*.

B6 105 — **least squares fit line** See *regression line*.

left-skewed (boxplot) See *skewed (boxplot).* **D11 15**

less than (<) A number is less than another number if it lies to the left of that number on the *number line.* For example, $-5 < -2$. **A2 103** *(30)*

like terms *Terms* that are the same except possibly for the *coefficients* (e.g. $1.4pqr$ and $0.7pqr$ are like terms). **B5 16**

line A straight line that extends infinitely far in both directions. Sometimes used as shorthand for *line segment* when no confusion can arise. **C8 9**

line of symmetry If a shape looks the same when it is reflected in a (mirror placed on a) *line* through the shape, then the line is called a line of symmetry, reflection line, mirror line or axis of symmetry. **C8 28**

line segment A finite (unbroken) section of a *line.* **C8 9**

linear equation in one unknown An *equation*, such as $3x - 2 = 4(2 + x)$, in which after *expanding* any brackets or fractions, each term is either a constant or a number times the *unknown* (in particular, there are no x^2 or x^3 terms). **B5 37** *(35)*

linear expression An *expression* of the form $mx + c$, where m and c are constants with $m \neq 0$ and x is a variable or unknown. **B6 104**

linear function A *function* with a rule of the form $y = mx + c$, where m and c are constants with $m \neq 0$. **B6 104**

linear regression The process of fitting a straight line as closely as possible to a given set of paired data. **D13 129**

linear relationship Two related quantities are said to have a linear relationship if the graph of one against the other is a straight line. **B6 67**

limit In the context of *inequalities*, a number that provides a restriction, or limitation, on the value of a *variable*. **A2 103** *(30)*

location A single number that represents an 'average', 'typical' or 'central' value of a *dataset.* **A4 193** *(33)*

logarithm The logarithm to base b of a number x, denoted by $\log_b x$, is the power to which b has to be raised to obtain x. For example, $2^4 = 16$ so $\log_2 16 = 4$. **D13 149** and **D13 154** *(48)*

logarithmic function A *function* whose rule is of the form $y = \log_b x$. **D13 169** *(48)*

logarithmic scale (to base 10) A scale (such as the Richter scale) in which each increase by 1 on the scale corresponds to a ten-fold increase in the quantity. The values on the scale are proportional to the logarithms to base 10 of the quantities that they represent. **D13 153**

lower quartile (Q1) See *quartiles.* **A4 202** *(33)*

lowest common multiple (LCM) (of integers) The lowest common multiple of two or more *integers* is the smallest positive integer that is a *multiple* of them all. **A3 121** *(31)*

lowest terms A numerical *fraction* is in its lowest terms or simplest form when it has been *cancelled* to give an integer *numerator* and integer *denominator* of smallest possible magnitude. **A1 34** *(29)*

magnitude (of a number) The value of the number without its sign, if it has one. For example, the magnitude of 3 is 3, and the magnitude of -3 is also 3. The magnitude of a number is often referred to as its size, modulus or absolute value. **B6 74**

manipulating See *rearranging (an equation)* and *rearranging (an expression).* **B5 12, 42** (34, 37)

A2 69 *(29)*	**map scale** The relationship between a distance on the map and the corresponding distance on the ground. It is often indicated by a graduated line, or as a ratio such as $1:500\,000$. See also *scale factor (of a map)*.
A2 66 *(30)*	**mathematical model** A collection of assumptions and mathematical statements that attempts to describe how some aspect of the real world behaves, and to make some predictions about its behaviour.
C10 172 *(44)*	**maximisation problem** The problem of finding the maximum value of a quantity and the circumstances under which it is obtained.
A4 196 *(33)*	**mean** See *arithmetic mean*.
A4 196 *(33)*	**median** When the values in a *dataset* are arranged in increasing (or decreasing) order, the median is the middle value if the number of values is odd, and the mean of the middle two values if the number of values is even.
C10 179 *(44)*	**minimisation problem** The problem of finding the minimum value of a quantity and the circumstances under which it is obtained.
A1 35	**mixed number** A number that consists of a whole number plus a *proper fraction*, such as $1\frac{2}{3}$.
A2 79 *(30)*	**modelling cycle** The process of designing a *mathematical model* by clarifying a problem, making assumptions to simplify it, describing it mathematically in order to obtain results, and using the results to refine the assumptions.
B6 74	**modulus (of a number)** See *magnitude (of a number)*.
C9 106	**multiple (of an expression)** An expression that has the original expression as a *factor*.
A3 121	**multiple (of a number)** A multiple of a number is the result of multiplying it by an *integer*. For example, $\ldots -12, -6, 0, 6, 12, 18, \ldots$ are multiples of 6. (Sometimes, such as when the natural numbers are being considered, the word 'multiple' is used to refer to positive multiples only.)
D13 118	**multiplication factor** Another name for *scale factor (of exponential change)*.
B5 25 *(34)*	**multiplier** An *expression* by which a bracketed expression is multiplied. For example, in $3xy(2+x^2)$ the multiplier is $3xy$.
B5 25 and **C9 82** *(34, 40)*	**multiplying out the brackets** The process of multiplying terms in brackets by a multiplier or by terms in other brackets to obtain an equivalent expression in which the brackets are not present.
C10 141	**n-shaped (parabola)** A *parabola* that is the opposite way up from the graph of $y = x^2$, i.e. its *vertex* is its highest point.
D13 155	**natural logarithm** The natural logarithm of a number x is the power to which e has to be raised to obtain x (that is, it is the logarithm of x to base e). The notation 'ln' is usually used in place of '$\log_e$'.
A1 44	**natural number** Any one of the counting numbers $1, 2, 3, 4, \ldots$.
B6 108	**negative correlation** The quantities in a set of *paired data* are said to have a negative correlation if one of the quantities tends to decrease as the other increases. In such cases the *correlation coefficient* is negative and the *regression line* has a negative *gradient*.
C8 60	**net (of a solid)** A *two-dimensional shape* that can be folded to obtain the surface of the *solid*.
C8 23	**nonagon** A *polygon* with nine sides.

null hypothesis The assumption that a phenomenon under investigation does not exist.	**D11 36**
number line A representation of the *real numbers* on a line in which the numbers become larger from left to right. In particular, all positive numbers lie to the right of zero and all negative numbers lie to the left of zero.	**A1 28**
numerator The top number or expression in a numerical or algebraic *fraction*.	**A1 33** and **B5 30** *(29, 32)*
oblong number A number given by the *expression* $n(n+1)$ for some *natural number* n. Each such number is double a *triangular number*.	**C9 78**
obtuse angle An *angle* greater than 90° and less than 180°.	**C8 10**
octagon A *polygon* with eight sides.	**C8 23**
odd (integer) An odd *integer* is one that is not divisible by 2.	**A1 45**
opposite (angle and side) In a triangle, a side and angle are opposite if the side is not one of the two line segments that form the angle.	**D12 60**
opposite angles (between two lines) *Angles* between two intersecting *lines*, that are opposite each other. These angles are equal.	**C8 13** *(38)*
opposite angles (in a quadrilateral) Two *angles* in a *quadrilateral* that do not have a side in common.	**C8 23** *(38)*
order (of a rotational symmetry) See *rotational symmetry*.	**C8 27**
origin The point with *coordinates* $(0,0)$.	**A2 85**
oscillation (of a sinusoidal curve) Any section of the graph of a *sinusoidal function* whose width on the x-axis is the *period* of the function.	**D14 222**
outliers One or more data values in a *dataset* that are considerably smaller or larger than the others.	**A4 189** *(33)*
paired data Lists of data values for two different variables that occur in pairs, such that for each item in one list there is a corresponding item in the other list. For example, the heights and weights of a group of people can form a set of paired data: each person's height would be paired with their weight.	**A4 191**
parabola The shape of the *graph* of any *equation* of the form $y = ax^2 + bx + c$, where a, b and c are constants with $a \neq 0$.	**C10 129** *(43)*
parabolic (curve) A curve whose shape is all or part of a *parabola* is said to be parabolic.	**C10 129**
parallel Two lines in a *plane* are parallel if they do not cross. This happens if either both the lines are vertical or they both have the same *gradient*. Similarly, a line and a plane, or two planes, are parallel if they do not cross.	**B6 78**
parallelogram A *quadrilateral* with opposite sides equal and *parallel*.	**C8 24** *(38, 39)*
pentagon A *polygon* with five sides.	**C8 23**
percent Means 'per 100' and is denoted by %. For example, $7\% = \frac{7}{100}$.	**A1 37** *(29)*
perfect correlation The quantities in a set of *paired data* are said to have a perfect correlation if all the *data points* lie on the *regression line*. In such cases the *correlation coefficient* is either 1 or −1.	**B6 108**
perfect square A *quadratic expression* that is equivalent to one of the form $(ax+b)^2$, where a and b are constants. Also, another name for a *square number*.	**C9 95** and **A3 153** *(40)*

C8 48	**perimeter** The boundary of a shape, or the length of the boundary.
D12 95 and **D14 222**	**period** A *function* and its *graph* are periodic, with period p units, if p is the smallest positive number for which the shape of the graph repeats itself every p units to the right or left. For example, the *sine* function has period 360°.
C8 47	**perpendicular** A line at right angles to a given line.
C8 49 *(39)*	**perpendicular height** The height of a shape when measured along a *line* at right angles to a side (or face) of the shape chosen to be the *base*.
D14 186	**pictogram** A representation of data by means of pictures. For example, copies of a picture suggestive of a topic might be stacked one above the other to form the bars of a chart similar to a *bar chart*, or they may each be labelled by a data item and have a size that represents a value associated with the item, such as its frequency.
C8 9	**plane** A flat surface that extends infinitely far in all directions.
C8 48	**plane shape** A shape that can be drawn in a *plane*.
C8 9	**point** A point has position but no size.
C8 23	**polygon** A *plane shape* with straight sides.
D14 215	**polyhedron** A solid with flat faces.
B6 108	**positive correlation** The quantities in a set of *paired data* are said to have a positive correlation if one of the quantities tends to increase as the other increases. In such cases the *correlation coefficient* is positive and the *regression line* has a positive *gradient*.
A3 131 *(32)*	**power** To raise a number to a power that is a positive *integer*, multiply it by itself the number of times specified by the power. For example, $2^3 = 2 \times 2 \times 2$. A number can also be raised to a power that is not a positive integer. The meaning of this operation, for powers that are rational numbers, is defined by the *index laws* $a^0 = 1$, $a^{-n} = 1/a^n$ and $a^{m/n} = (\sqrt[n]{a})^m$. See also *index form*.
A1 23	**precision (of an answer)** How many *significant figures* the answer is stated to.
A4 213	**precision (of a set of measurements)** How close the measurements in a set of repeated measurements are to each other.
A4 183	**primary data** *Data* that you collect yourself.
A3 124	**prime (number)** A *natural number* that has exactly two *factors* (itself and 1).
A3 128 *(31)*	**prime factorisation** The prime factorisation of a *natural number* is the *product* of prime *factors* that is equal to it.
C8 58 *(40)*	**prism** A *three-dimensional shape* formed by filling in the space between two parallel *congruent* polygons. These polygons are faces of the prism and they are congruent to all parallel cross-sections through the prism. The edges joining the vertices of one of the polygons to the matching vertices of the other polygon are perpendicular to the polygons.
D14 181 *(49)*	**probability** A measure of how likely something is to occur. A probability can be expressed as a fraction, as a decimal, as a percentage or in the form of an x in y chance. For example, $\frac{1}{200}$, 0.005 and 0.5% are all ways to describe a 1 in 200 chance.
A1 14 *(28, 31)*	**product** The product of two or more numbers is the result of multiplying them.

projectile An object that is propelled through space by a force that ceases after launch. C10 132

proof A demonstration that a piece of mathematics always works. B5 7

proper fraction A numerical *fraction* in which the magnitude of the *numerator* is smaller than that of the *denominator*, such as $\frac{2}{3}$. A1 35

proportional See *direct proportion*. B6 87, 88

pseudo-random (numbers) *Random numbers* generated by a computer *algorithm*. D11 29

Pythagorean triple Three whole numbers such that the square of one of them is equal to the *sum* of the squares of the other two. For example, the numbers 3, 4, and 5 form a Pythagorean triple, since $5^2 = 3^2 + 4^2$. C8 46 *(39)*

quadrant The x- and y-*axes* divide the *plane* into four regions known as quadrants. The quadrant between the positive x- and y-axes is called the first quadrant, followed (anticlockwise) by the second, third and fourth quadrants. D12 88

quadratic (expression) An *expression* of the form $ax^2 + bx + c$, where a, b and c are constants with $a \neq 0$, is called a quadratic expression in x, or a quadratic in x, or just a quadratic. C9 87 *(41)*

quadratic equation Any *equation* that can be expressed in the form $ax^2 + bx + c = 0$ (by *rearranging* if necessary) is called a quadratic equation in x. In this equation, x is an *unknown*, and a, b and c are constants with $a \neq 0$. C9 88 *(42, 43, 44)*

quadratic formula A *formula* that gives the *solutions* of a *quadratic equation*. C10 152 *(43)*

quadratic function A *function* whose rule is of the form $y = ax^2 + bx + c$, where a, b and c are constants with $a \neq 0$. C10 138 *(43)*

quadratic model A *mathematical model* based on a *formula* of the form $y = ax^2 + bx + c$, where a, b and c are constants with $a \neq 0$. C10 130

quadrilateral A *polygon* with four sides. C8 23

quartiles When the values in a *dataset* are arranged in ascending order, the lower quartile is the *median* of the values in the lower half of the dataset, and the upper quartile is the median of the values in the upper half of the dataset (with, in each case, the middle value thrown out if the number of values is odd). A4 202 *(33)*

quotient A quotient of two numbers is the result of dividing one by the other. A1 14 *(28, 31)*

radian A unit used to measure *angle*. One radian is the angle *subtended* at the centre of a *circle* by an *arc* that is the same length as the *radius*. A full turn is 2π radians. D12 102 *(46)*

radius A *line segment* from the centre to the *circumference* of a *circle*, or the length of such a line segment. C8 54 *(39)*

radius (of a circular arc) The *radius* of the *circle* on whose *circumference* the *arc* lies. D12 104

random numbers Short for *uniform random numbers* (unless the context indicates that the numbers are not equally likely to occur). D11 28 *(45)*

range The range of a *dataset* is the *difference* between its largest and smallest values. A4 200

B6 79 **rate of change** The *gradient* of a straight-line *graph* tells you the amount by which the *variable* on the *vertical axis* increases when the variable on the *horizontal axis* increases by one unit. This increase in the first variable (which is actually a decrease if negative) is known as the rate of change of the first variable with respect to the second.

A3 159 *(32)* **ratio** A ratio of two or more quantities specifies how many parts of each quantity there are. For example, the ratio $1:2:4$ (read 1 to 2 to 4) means that there is 1 part of the first quantity for every 2 parts of the second quantity and every 4 parts of the third quantity. A ratio of two quantities is sometimes written as a *fraction* or decimal. For example, $3:2$ can be written as $\frac{3}{2}$ or as 1.5.

A3 136 **rational number** A number that can be written as an *integer* divided by an integer.

A3 150 **real line** Another name for the *number line*, since each point on the number line represents a *real number*.

A3 150 **real numbers** The set of all the *rational numbers* together with all the *irrational numbers*. Each real number is represented by a point on the *number line*.

B5 42 *(35)* **rearranging (an equation)** Rearranging (or manipulating) an *equation* is the process of: doing the same thing to both sides of the equation; *rearranging the expressions* in the equation; or swapping the sides of the equation.

B5 12 *(34)* **rearranging (an expression)** The process of writing an *expression* in a different way to obtain an *equivalent expression* is known as rearranging, manipulating or rewriting the expression.

A3 142 **reciprocal** The reciprocal of a number is 1 divided by the number.

C8 24 *(39)* **rectangle** A *quadrilateral* with four *right angles*.

A3 137 **recurring decimal** A decimal number with a block of one or more *digits* after the decimal point that repeats indefinitely.

C8 28 **reflection line** See *line of symmetry*.

C8 10 **reflex angle** An *angle* greater than 180° and less than 360°.

B6 105 **regression line** The straight line on a scatterplot of *paired data* that 'best' fits the data. Other names for this line include least squares fit line, best fit line and trend line.

C8 25 **regular (polygon)** A *polygon* with equal sides and equal *interior angles*.

A1 39 **relative comparison** A relative comparison is one in which proportions are used, whereas an absolute comparison is one in which differences are used. For example, if the values of A and B are 5 and 10, respectively, then the statement 'B is twice as big as A' is a relative comparison, whereas the statement 'B is 5 units larger than A' is an absolute comparison.

B5 25 *(34)* **removing the brackets** See *multiplying out the brackets*.

C9 96 *(44)* **repeated solution** If the two *solutions* of a *quadratic equation* are the same, then the equation is said to have a repeated solution.

A1 48 **result** See *theorem*.

B5 12 *(34)* **rewriting an expression** See *rearranging an expression*.

rhombus A *parallelogram* with four equal sides. **C8 24**

right angle An *angle* of 90°. **C8 10**

right-angled triangle A triangle in which one angle is equal to 90°. **C8 19**

right-skewed (boxplot) See *skewed (boxplot)*. **D11 15**

rise Given two points (x_1, y_1) and (x_2, y_2), with $x_1 < x_2$, the rise is $y_2 - y_1$. **B6 70** *(35)*

risk The *probability* of an undesirable event occurring. **D14 181**

root mean squared deviation (RMS) Another name for *standard deviation*. **A4 205** *(34)*

rotational symmetry A shape has rotational symmetry if it can be rotated through a fixed *angle* (less than a full turn) about a fixed *point* to produce a rotated shape that looks the same as the original shape. If there are, say, three positions in which the shape looks the same, then the shape is said to have rotational symmetry of order 3, or three-fold rotational symmetry. **C8 27**

rounding error An error in an answer resulting from rounding performed at an earlier step of the calculation. **A1 24**

run Given two points (x_1, y_1) and (x_2, y_2), with $x_1 < x_2$, the run is $x_2 - x_1$. **B6 70** *(35)*

satisfying (an equation) See *solution (of an equation in one unknown)*. **B5 36** *(35)*

satisfying (an inequality) A value of a *variable* for which an *inequality* is true is said to satisfy the inequality. **A2 105** *(30)*

satisfying (simultaneous equations) See *solution (of simultaneous equations)*. **B7 146** *(37)*

scale factor (of exponential change) If a quantity is subject to discrete *exponential change*, then from an initial starting number each subsequent value is obtained by multiplying its predecessor by a *constant*. This constant is known as the scale factor. More generally, if a variable y changes exponentially with respect to x (continuously or discretely), then whenever the value of x increases by a fixed number of units, the value of y is multiplied by a constant. This constant is called the scale factor over that number of units. In particular, if the relationship between x and y is given by the equation $y = ab^x$, then b is the scale factor over 1 unit (and a is the starting number). **D13 118, 121, 134, 137** *(47)*

scale factor (of an image) The number by which distances on an image are multiplied when the image is enlarged or reduced. **A3 165**

scale factor (of a map) The number by which a distance on the map has to be multiplied to obtain the actual distance on the ground. **A2 69** *(29)*

scalene triangle A triangle all of whose sides are different lengths. **C8 20**

scatterplot A *graph* on which *paired data* are plotted. **B6 68**

scientific notation A notation in which a number is written as a decimal number between 1 and 10 (including 1, but excluding 10), multiplied by a power of 10; for example, 1.92×10^{-2} and 9.994×10^{30}. **A3 145** *(32)*

secondary data Existing *data* that you can use or adapt for your purpose. **A4 183**

sector The shape enclosed by an *arc of a circle* together with the two *radii* from the endpoints of the arc. **C8 54**

C8 54 **segment** The shape enclosed by an *arc of a circle* and the *chord* joining the ends of the arc.

C8 55 **semicircle** The shape enclosed by a *diameter* of a *circle*, together with an *arc* from one end of the diameter to the other.

D12 86 *(46)* **semi-perimeter (of a shape)** Half of the *perimeter* of the shape.

C9 75 *(40)* **sequence** A list, usually of numbers.

D12 87 **sign of an angle** This indicates the direction of rotation about a *point*. Positive *angles* correspond to anticlockwise rotations, and negative angles correspond to clockwise rotations.

A1 20 *(28)* **significant figures (s.f.)** The first significant figure in a number is the first non-zero digit when reading the number from left to right, the second significant figure is the digit immediately to the right of this digit and so on.

C8 29 *(39)* **similar** Geometric *figures* that have the same shape (flipped if necessary), but not necessarily the same size, are said to be similar.

A1 34 *(29)* **simplest form (of a fraction)** Another name for *lowest terms*.

A3 160 *(32)* **simplest form (of a ratio)** A *ratio* is in its simplest form if the numbers in the ratio are whole numbers with no positive *common factor* other than 1.

B5 12 *(34)* **simplifying an expression or equation** The process of *rearranging an expression* or *equation* to make it simpler.

D11 31 **simulation (using random numbers)** The process of using random numbers to investigate statistical features that may or may not occur by chance.

B7 143 *(37)* **simultaneous equations** Two or more *equations* that apply to the *unknowns* simultaneously.

D12 61, 90 *(45)* **sine** The sine of an *angle* θ, written $\sin\theta$, is the y-coordinate of the *point* obtained by rotating the point $(1, 0)$ about the *origin* through the angle θ. For an acute angle θ in a *right-angled triangle*, $\sin\theta$ is equal to the length of the side *opposite* θ divided by the length of the *hypotenuse*.

D12 94 **sine curve** The graph of the *sine* function.

D12 77 *(45)* **Sine Rule** A rule for *solving a triangle* in which a side and its opposite angle, together with at least one other angle or side, are known.

D14 222 *(50)* **sinusoidal curve** A curve that can be obtained by shifting, stretching or compressing the graph of the *sine* function horizontally or vertically.

D14 222 *(50)* **sinusoidal function** A *function* whose graph is a *sinusoidal curve*. All general sine functions and all general cosine functions are sinusoidal functions. Moreover, every sinusoidal function can be expressed as either a general sine function or as a general cosine function.

B6 74 **size (of a number)** See *magnitude (of a number)*.

D11 15 **skewed (boxplot)** A *boxplot* is left-skewed if the data values are more sparsely spread at the left and more densely concentrated at the right of the boxplot. Likewise, a boxplot is right-skewed if the data values are more sparsely spread at the right and more densely concentrated at the left of the boxplot.

C8 59 *(40)* **slant height (of a cone)** The distance from the *apex* of the *cone* to a *point* on the *circumference* of its *base*.

slope See *gradient*. B6 69, 70, 78 *(35)*

solid Another name for a *three-dimensional shape*. C8 58 *(40)*

solution (of an equation in one unknown) Any value of the *unknown* that makes the two sides of the *equation* equal is said to satisfy the equation and is called a solution of the equation. The process of finding such a solution is known as solving the equation. B5 36 *(35)*

solution (of simultaneous equations) Values of the *unknowns* that simultaneously satisfy all the equations are together called a solution of the *simultaneous equations*. Such a solution is said to satisfy the equations. The process of finding a solution is known as solving the equations. B7 146 *(37)*

solving (an equation) See *solution (of an equation in one unknown)*. B5 36 *(35)*

solving (an inequality) The process of finding all the numbers that satisfy an *inequality*. B7 164

solving (simultaneous equations) See *solution (of simultaneous equations)*. B7 146 *(37)*

solving a triangle The process of calculating unknown lengths or *angles* in a *triangle*. D12 75, 83 *(46)*

speed A measure of how far an object travels in a particular period of time. See also *average speed*. A2 72 *(30)*

sphere A *three-dimensional shape* whose boundary consists of all *points* that are a fixed distance from a fixed point called the centre of the sphere. C8 61 *(40)*

spread (of a dataset) How widely the values in the *dataset* are distributed. A4 200 *(34)*

spurious precision The display of values to a greater-than-warranted number of *significant figures*. A4 189 *(33)*

square A *quadrilateral* with four equal sides and four *right angles*. C8 24

square (of a number) The square of a number is the result of multiplying it by itself. A3 131

square numbers The numbers $1, 4, 9, 16, \ldots$, obtained by multiplying each *natural number* by itself. A1 45

square root A square root of a number is a number that when multiplied by itself gives the original number. A3 151 *(32)*

standard deviation (SD) The standard deviation of a set of values is the *square root* of the *mean* of the squares of the deviations, where the deviations are the *differences* of each value from the mean. (Sometimes a slightly different definition is used – see page 34.) A4 205 *(34)*

standard form Another name for *scientific notation*. A3 145 *(32)*

starting number See *scale factor (of exponential change)*. D13 118 *(47)*

straight angle An *angle* of 180°. C8 10 *(38)*

strict inequality A statement involving one or more of the inequality signs $<$ or $>$, but not $\leq$ or $\geq$. A2 104

subject (of a formula) See *formula*. A2 89

subscript Characters such as the 1 and the 2 in $x_2 - x_1$ are known as subscripts; they are smaller and set slightly lower than normal. They are often used to distinguish distinct but related variables. B6 76

A2 89 **substituting** The process of replacing *variables* in an *expression* or *equation* with numerical values.

D12 102 **subtended (angle)** The *angle* between the *line segments* that join a *point* to each end of an *arc* is said to be the angle subtended by the arc at the point.

A1 14 *(28, 31)* **sum** The result of adding together two or more numbers.

A3 153 *(32)* **surd** A numerical expression containing one or more *irrational* roots of numbers.

C8 60 *(40)* **surface area (of a solid)** The area of the *solid's* surface.

D12 61, 91, 92 **tangent** The tangent of an *angle* θ, written $\tan\theta$, is defined by $\tan\theta = \sin\theta/\cos\theta$, provided that $\cos\theta \neq 0$. It is the y-coordinate of the *point* where the line $x = 1$ meets the line obtained by rotating the x-axis through the angle θ about the *origin*. For an acute angle θ in a *right-angled triangle*, $\tan\theta$ is equal to the length of the side *opposite* θ divided by the length of the side *adjacent* to θ.

B5 13 **term (of an expression)** In an *expression* formed by adding or subtracting a list of items, each item is called a term of the expression. A sign (plus or minus) at the start of a term is part of the term.

C9 75 *(40)* **term (of a sequence)** An entry (usually a number) in the *sequence*.

A3 137 **terminating decimal** A decimal number that has only a finite number of *digits* after the decimal point.

D13 147 ***the* exponential function** The *function* whose *rule* is $y = e^x$.

A1 48 **theorem** A mathematical statement that has been proved is called a theorem or result.

C8 58 *(40)* **three-dimensional (shape)** A shape that extends in three mutually perpendicular directions. In a sketch: width is extent across the page; height is extent up and down the page; and depth is extent into the page (using perspective).

A1 35 **top-heavy fraction** A numerical *fraction* in which the magnitude of the *numerator* is larger than that of the *denominator*, such as $\frac{5}{3}$.

C10 132 **trajectory** The path that a *projectile* follows.

C8 24 *(39)* **trapezium** A *quadrilateral* with one pair of opposite sides *parallel*.

B6 105 **trend line** See *regression line*.

D11 28 **trial** A single experiment or observation for which a number of outcomes are possible, but only one can occur at a time. For example, the tossing of a coin is a trial with an outcome of either 'head' or 'tail'.

D13 132 **trial and improvement** A way of 'homing in' on the solution of an equation by repeatedly trying values to see whether, at each stage, a larger or a smaller value would improve the approximation.

C9 74 **triangular number** A number given by the *expression* $\frac{1}{2}n(n+1)$ for some *natural number* n. Each such number is the number of dots that can be arranged in a triangular shape with 1 dot in the first row, 2 dots in the second row, and so on, up to n dots in the final row.

triangular prism A *prism* with triangular cross-section. **C8 58** *(40)*

trigonometric function A *function* whose rule takes an *angle* θ as input, and outputs one of the trigonometric values associated with that angle, such as $\sin\theta$, $\cos\theta$ or $\tan\theta$. **D12 93** *(47)*

trigonometric ratio (of an angle) The ratio of two sides of a right-angled triangle that contains the angle. **D12 61** *(45)*

trigonometry The branch of mathematics that is concerned with methods of using triangles to find unknown lengths and angles. **D12 58**

two-dimensional (shape) A shape that extends in two directions. **C8 58**

two-sample A two-sample dataset is one that consists of two sets of values of the same *variable*, allowing the two samples to be compared. **A4 191**

u-shaped (parabola) A *parabola* that is the same way up as the graph of $y = x^2$, i.e. its *vertex* is its lowest point. **C10 141**

uniform random numbers A sequence of numbers, each of which is selected uniformly (that is, with equal chance) and *independently* of its predecessors. **D11 31** *(45)*

unit circle The *circle* with *radius* 1 centred on the *origin*. **D12 88**

unknown A letter that represents a particular, though possibly unknown, number. **B5 13**

upper quartile (Q3) See *quartiles*. **A4 202** *(33)*

variable A letter used to represent different numbers. **A2 89**

variance The square of the *standard deviation*. **A4 205** *(34)*

vertex A *point* where two *line segments* meet. **C8 9**

vertex (of a parabola) The *point* at which the *parabola* intersects its *axis of symmetry*. **C10 138** *(43)*

vertical axis A vertical line with a scale that is used to specify the vertical position of a point. **A2 84** and **B6 64** *(35)*

vertical coordinate See *coordinates*. **A2 85** and **B6 64**

vertical displacement (of a sinusoidal curve) The mean of the maximum and minimum values of the curve. **D14 226** *(50)*

vertical intercept See *intercept*. **B6 83** *(35, 43, 48)*

volume (of a solid) The amount of space that the *solid* occupies. **C8 60** *(40)*

whiskers See *boxplot*. **D11 11** *(45)*

X angles An informal name for *opposite angles* between two *lines*. **C8 13** *(38)*

x-intercept A value on a graph's x-axis scale where the graph crosses or touches the axis, i.e. a value of x for which $y = 0$. **B6 82** and **C10 143**

y-intercept A value on a graph's y-axis scale where the graph crosses or touches the axis, i.e. a value of y for which $x = 0$. **B6 82** and **C10 143**

Z angles An informal name for *alternate angles*. **C8 14** *(38)*

4 Key skills and results

The following key skills and results have been collected from the units of MU123. They are usually listed in the order in which they occur in the module, each with a reference in the margin to its location.

Unit 1: Starting points

Book A, Unit 1, page 13

Using the BIDMAS rules

Carry out mathematical operations in the following order.

B	brackets	
I	indices (powers and roots)	
D	divisions	same precedence
M	multiplications	
A	additions	same precedence
S	subtractions	

When operations have the same precedence, work from left to right.

Book A, Unit 1, Example 2, page 16

Converting units

To convert from one unit to another, first find out how many of the smaller units are equivalent to one of the larger units.

- To convert to the smaller unit, multiply by this number.
- To convert to the larger unit, divide by this number.

Book A, Unit 1, page 19

Rounding a number

To avoid rounding errors, use full calculator precision throughout a calculation and round only the final answer.

- Look at the digit immediately after where you want to round.
- Round up if this digit is 5 or more, and down otherwise.

For example, $2.3971 = 2.40$ (to 2 d.p.) and $36.7972 = 36.80$ (to 4 s.f.).

Book A, Unit 1, page 29

Adding and subtracting negative numbers

- Adding a negative number is the same as subtracting the corresponding positive number. For example, $5 + (-2) = 5 - 2 = 3$.
- Subtracting a negative number is the same as adding the corresponding positive number. For example, $5 - (-2) = 5 + 2 = 7$.

Book A, Unit 1, page 31

The following table might help you to remember these rules:

	+	−
+	+	−
−	−	+

Multiplying and dividing negative numbers

When two numbers are multiplied or divided:

- If the signs are *different*, then the answer is *negative*. For example, $9 \div (-3) = -3$ and $(-3) \times 7 = -21$.
- If the signs are *the same*, then the answer is *positive*. For example, $-9 \div (-3) = 3$ and $3 \times 7 = 21$.

Writing a fraction in its simplest form

Book A, Unit 1, page 34

Keep cancelling the fraction until it is no longer possible to exactly divide both the numerator and the denominator by the same whole number (other than 1). The result will be an equivalent fraction in simplest form.

$$\frac{\cancel{24}^{\cancel{12}^{4}}}{\cancel{30}_{\cancel{15}_{5}}} = \frac{4}{5}$$

Calculating a fraction of a quantity

Book A, Unit 1, Example 10, pages 35–36

Multiply the fraction by the quantity.
For example, $\frac{5}{8}$ of $20 = \frac{5}{8} \times 20 = 5 \div 8 \times 20 = 12.5$.

Converting a percentage to a fraction or decimal

Book A, Unit 1, page 37

First write the percentage in the form of a fraction with denominator 100, then simplify to obtain a fraction, or divide out to obtain a decimal.
For example, $45\% = \dfrac{45}{100} = \dfrac{9}{20}$ and $45\% = \dfrac{45}{100} = 45 \div 100 = 0.45$.

Converting a fraction or decimal to a percentage

Book A, Unit 1, page 37

Multiply the fraction or decimal by 100% $(= 1)$.
For example, $\dfrac{2}{5} = \dfrac{2}{5} \times 100\% = 40\%$ and $0.015 = 0.015 \times 100\% = 1.5\%$.

Expressing a number as a percentage of another number

Book A, Unit 1, page 38

Calculate $\dfrac{\text{first number}}{\text{second number}} \times 100\%$.

Calculating a percentage of a quantity

Book A, Unit 1, page 40

Change the percentage to a fraction or a decimal, and multiply by the quantity.
For example, 2.5% of $450 = \dfrac{2.5}{100} \times 450 = 0.025 \times 450 = 11.25$.

Calculating a percentage increase or decrease

Book A, Unit 1, page 40

Calculate $\dfrac{\text{actual increase or decrease}}{\text{original value}} \times 100\%$.

Calculating the value resulting from a percentage change

Book A, Unit 1, pages 41–42

Change 100% by the required percentage and multiply the resulting adjusted percentage by the value. For example, if 599 is decreased by 15%, the new value is 85% of $599 = 0.85 \times 599 = 509.15$. If 800 is increased by 5%, the new value is 105% of $800 = 1.05 \times 800 = 840$.

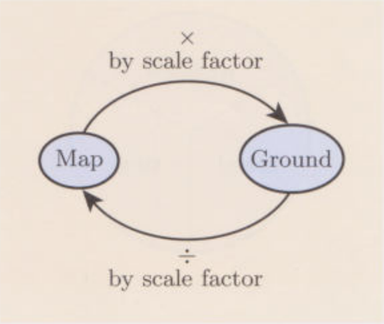
Unit 2: Mathematical models

Using the scale factor of a map to convert distances

Book A, Unit 2, Example 2, pages 70–71

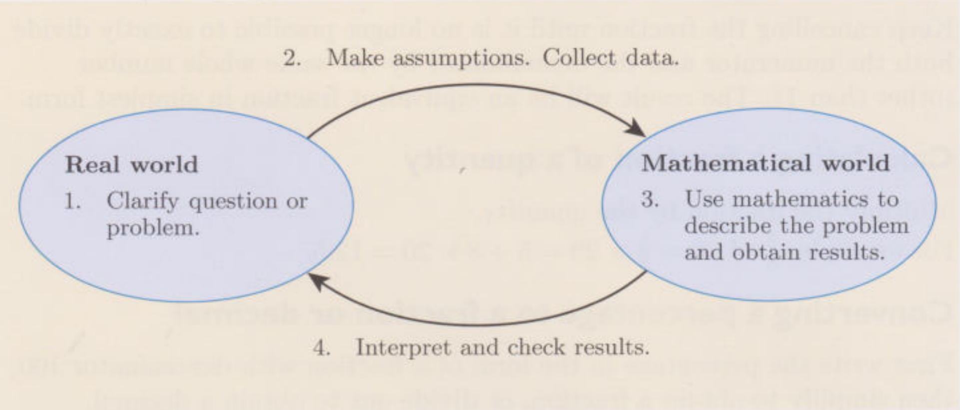
If not already known, deduce the scale factor from the map scale. (For instance, the scale 1 : 500 000 means that the scale factor is 500 000, as does the scale '2 cm represents 10 km'.)

- *To calculate a ground distance*: multiply the map distance by the scale factor of the map and express the result in the required units.
- *To calculate a map distance*: divide the ground distance by the scale factor of the map and express the result in the required units.



Book A, Unit 2, page 79

The modelling cycle



Book A, Unit 2, page 84

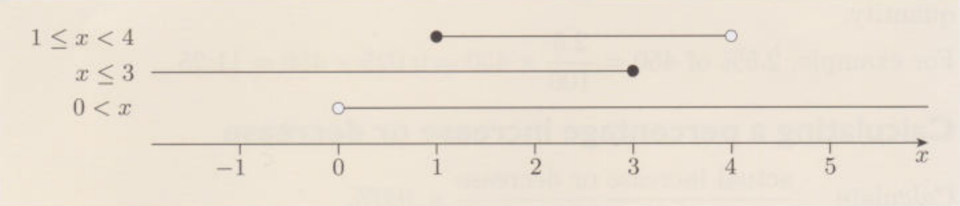
Drawing a graph or chart based on data

- Include a clear title and the source of the data.
- Label the axes with the names of the quantities and (if applicable) the units.
- Mark the scales clearly, choosing scales that are easy to interpret and that make good use of the space available.

Book A, Unit 2, Example 15, page 106

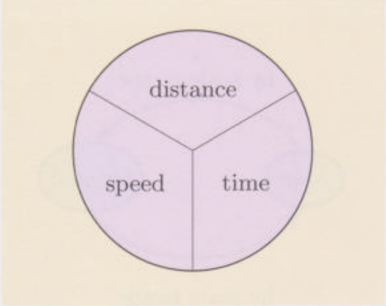
Using inequalities to specify the values taken by a variable

In many cases the values taken by a variable form an interval. The following examples, concerning a variable x, illustrate how inequalities are used to describe such intervals.



- An inequality such as $x < 4$ constrains x to lie to the left of a limit (here 4).
- An inequality such as $0 < x$ constrains x to lie to the right of a limit (here 0).
- To include the limit in the constraint, use an inequality that is not strict ($\leq$ or $\geq$).
- Diagrammatically, inclusion of a point is indicated by a solid circle and exclusion is indicated by an empty circle.

Book A, Unit 2, page 109



Using the speed, distance and time formulas

The three formulas relating the distance, average speed and time for a journey can all be recalled from the diagram in the margin. They are:

$$\text{speed} = \frac{\text{distance}}{\text{time}}, \quad \text{time} = \frac{\text{distance}}{\text{speed}}, \quad \text{distance} = \text{speed} \times \text{time}.$$

Unit 3: Numbers

Finding the factors of a natural number (factorisation)

Book A, Unit 3, page 122

1. Try the numbers $1, 2, 3, 4, \ldots$ in turn. Whenever you find a factor, write down the other factor in the factor pair.
2. Stop when you get a factor pair that you have already.

Testing for divisibility

Book A, Unit 3, page 123

A natural number is divisible by

- 2 if it ends in 0, 2, 4, 6, or 8
- 3 if its digits add up to a multiple of 3
- 5 if it ends in 0 or 5
- 9 if its digits add up to a multiple of 9.

If a number does not satisfy a test above, then it is not divisible by the specified number.

The fundamental theorem of arithmetic

Book A, Unit 3, page 128

Every natural number greater than 1 can be written as a product of prime numbers in just one way (except that the order of the primes in the product can be changed).

Obtaining a prime factorisation of a composite number

Book A, Unit 3, pages 128-129

Proceed step by step as follows:

1. Divide the number by its smallest prime factor.
2. Decide whether the result is a prime; if not, divide it by its smallest prime factor.
3. Repeat the previous step until the result produced is a prime.

The example in the margin shows a handy way to record the steps. The required factorisation appears on the final line.

$$\begin{aligned} 252 &= 2 \times 126 \\ &= 2 \times 2 \times 63 \\ &= 2 \times 2 \times 3 \times 21 \\ &= 2 \times 2 \times 3 \times 3 \times 7 \\ &= 2^2 \times 3^2 \times 7. \end{aligned}$$

Finding the LCM or HCF of two or more natural numbers

Book A, Unit 3, page 130

- Find the prime factorisations of the numbers.
- To find the LCM, multiply together the highest power of each prime factor occurring in any of the numbers.
- To find the HCF, multiply together the lowest power of each prime factor common to all the numbers.

Adding or subtracting fractions

Book A, Unit 3, page 139

1. Make sure that the denominators are the same. (You may need to use a common denominator to write each fraction as an appropriate equivalent fraction.)
2. Add or subtract the numerators.
3. Write the answer in its simplest form.

Multiplying fractions

Book A, Unit 3, page 140

Multiply the numerators together and multiply the denominators together. Write the answer in its simplest form.

Book A, Unit 3, page 142

Dividing by a fraction

Multiply by its reciprocal (obtained by swapping over its numerator and denominator). Write the answer in its simplest form.

Book A, Unit 3, page 145

Expressing a number in scientific notation

1. Place a decimal point between the first and second significant digits to give a number between 1 and 10.
2. Count to find the power of 10 by which this number should be multiplied (or divided) to restore it to the original number.

Book A, Unit 3, pages 153–156

Simplifying surds

- Simplify roots of integers with square factors (e.g. $\sqrt{12} = 2\sqrt{3}$).
- Simplify products and quotients of roots (e.g. $\sqrt{15}/\sqrt{3} = \sqrt{5}$).
- Add or subtract (multiples of) roots that are the same (e.g. $\sqrt{12} + 3\sqrt{3} = 5\sqrt{3}$).

Book A, Unit 3, page 159

Index laws (rules for powers)

Here the numbers must be appropriate for the operations. For example, you cannot divide by zero, or take a square root of a negative number.

$$a^m \times a^n = a^{m+n}, \qquad \frac{a^m}{a^n} = a^{m-n}, \qquad (a^m)^n = a^{mn}$$

$$(a \times b)^n = a^n \times b^n, \qquad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$a^0 = 1, \qquad a^{-n} = \frac{1}{a^n}$$

$$a^{\frac{1}{n}} = \sqrt[n]{a}, \qquad a^{\frac{m}{n}} = (\sqrt[n]{a})^m$$

Book A, Unit 3, page 160

Finding a ratio equivalent to a given ratio

Multiply or divide each number in the ratio by the same non-zero number. If possible, a ratio is usually written in its simplest form, where the numbers are integers without a common factor greater than 1.

Book A, Unit 3, Activity 37, page 161

Comparing ratios of two numbers

Write each ratio in the form of a fraction (or in the form 'number : 1') and compare the fractions (or the numbers to the left of the colons).

Book A, Unit 3, Example 16, page 161

Finding an approximate ratio

1. Write the ratio in the form 'number : 1'.
2. Replace the number by a simple fraction that approximates the number.
3. Simplify the resulting ratio.

Book A, Unit 3, Example 17, page 162

Dividing a quantity in a ratio

1. Calculate the sum of the numbers in the ratio.
2. To find the portion of the quantity corresponding to each ratio number, divide the number by the sum and multiply by the quantity.

For example, since $5 + 2 + 3 = 10$, the ratio $5 : 2 : 3$ divides 1250 into $\frac{5}{10} \times 1250 = 625$, $\frac{2}{10} \times 1250 = 250$ and $\frac{3}{10} \times 1250 = 375$, respectively.

Unit 4: Statistical summaries

The four stages of a statistical investigation (PCAI)

Book A, Unit 4, page 180

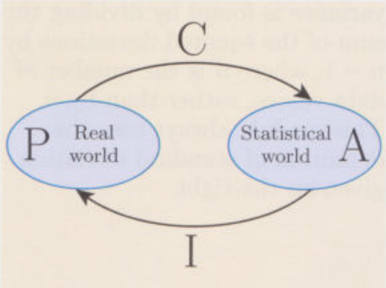
There are four clearly identifiable stages in most statistical investigations, which can be summarised as follows.

Stage 1 **P**ose a question

Stage 2 **C**ollect relevant data

Stage 3 **A**nalyse the data

Stage 4 **I**nterpret the results



The problem starts in the real world and is resolved by making a journey into the statistical world and back again. Complete resolution of the problem might require several trips around the cycle.

Scanning a dataset

Book A, Unit 4, page 195

Before performing a detailed analysis of any dataset, it is advisable to examine the values to see if any patterns or anomalies stand out. For example, you might look for

- missing data
- spurious precision
- dubious data, perhaps caused by a misplaced decimal point
- coded values, perhaps to signal 'value missing'
- constraints such as that the data ought to lie between 0 and 100
- the presence of outliers.

Finding the mean of a dataset

Book A, Unit 4, page 196

To find the mean of a set of numbers, add all the numbers together and divide by however many numbers there are in the set.

Finding the median of a dataset

Book A, Unit 4, page 197

To find the median of a set of numbers:

- Sort the data into increasing (or decreasing) order.
- If there is an odd number of data values, the median is the middle value.
- If there is an even number of data values, the median is the mean of the middle two values.

Finding the quartiles and the interquartile range of a dataset

Book A, Unit 4, page 204

1. Arrange the dataset in increasing order.
2. Next:
   (a) If there is an even number of data values, then the lower quartile (Q1) is the median of the lower half of the dataset, and the upper quartile (Q3) is the median of the upper half of the dataset.
   (b) If there is an odd number of data values, throw out the middle data value (which of course has the median value of the dataset). Then the lower quartile (Q1) is the median of the lower half of the new dataset, and the upper quartile (Q3) is the median of the upper half of the new dataset.
3. The interquartile range (IQR) is Q3 − Q1.

Book A, Unit 4, page 205

In some circumstances a slightly different definition of standard deviation is used, in which the variance is found by dividing the sum of the squared deviations by $n-1$, where n is the number of data values, rather than by n. This module always uses the definition of standard deviation given on the right.

Finding the standard deviation of a dataset

1. Find the mean of the dataset.
2. Find the difference of each data value from the mean – these are the 'deviations', often labelled the d values.
3. Square each deviation – this gives the d^2 values.
4. Find the mean of these squared deviations – this number is the 'mean squared deviation', better known as the variance.
5. Find the square root of the variance to get the 'root mean squared deviation' – that is, the standard deviation.

Unit 5: Algebra

Book B, Unit 5, page 24

Simplifying an expression

1. Identify the terms.
2. Simplify each term, remembering to include the sign (plus or minus) at the start of each term. (If the term includes brackets or algebraic fractions, consider also whether to multiply out the brackets or expand the fractions, but remember to simplify any resulting new terms.)
3. Collect any like terms.

Book B, Unit 5, page 23

Identifying the terms in an expression

Use the fact that each term after the first starts with a plus or minus sign that is not inside brackets. Thus $5ab + 6c^2g\sqrt{2} - (-4d) + (-a)(-3+x)d$ has terms $+5ab$, $+6c^2g\sqrt{2}$, $-(-4d)$ and $+(-a)(-3+x)d$.

Book B, Unit 5, page 22

Remember: a plus times a plus gives a plus; a plus times a minus gives a minus. Also: adding/subtracting the negative of something is the same as subtracting/adding the something.

Simplifying a term in an expression

1. Find the overall sign and write it at the front.
2. Simplify the rest of the coefficient and write it next.
3. Write any remaining parts of the term in some appropriate order; for example, letters are usually ordered alphabetically. Use index notation to avoid writing letters (or other parts) more than once.

For example, $-(-2pq) \times (-3qp^2) = -6p^3q^2$.

Book B, Unit 5, page 26

Multiplying out brackets

Multiply each term inside the brackets by the multiplier. Simplify each product as you multiply out. For example, $2a(3a+2b) = 6a^2 + 4ab$.

See the Unit 9 entries for how to multiply out expressions with more than one pair of brackets, such as $(a+b)(c+d)$.

Book B, Unit 5, page 29

Removing brackets with a plus or minus sign in front

- If the sign is plus, keep the sign of each term inside the brackets the same. For example, $+(a^2 + 3ab - d) = +a^2 + 3ab - d$.
- If the sign is minus, change the sign of each term inside the brackets. For example, $-(a^2 + 3ab - d) = -a^2 - 3ab + d$.

Book B, Unit 5, page 31

Expanding an algebraic fraction

Divide each term of the numerator by the denominator. Simplify each quotient as you divide through. For example, $\dfrac{10x + x^2 - 8}{x} = 10 + x - \dfrac{8}{x}$.

Collecting like terms

Book B, Unit 5, page 17

Replace each group of like terms by a single term whose coefficient is the sum of the coefficients of the terms in the group. For example, $a + 5xy + a - 2yx - 2a = 3xy$.

Solving a linear equation in one unknown

Book B, Unit 5, page 45

Carry out a sequence of steps. In each step, do one of the following:

- do the same thing to both sides
- simplify one side or both sides
- swap the sides.

Remember that you have to do the same thing to the *whole* of each side, namely one of:

- add something
- subtract something
- multiply by something
- divide by something non-zero.

Aim to do the following, in order.

1. Clear any fractions and multiply out any brackets. To clear fractions, multiply both sides by a suitable number.
2. Add or subtract terms on both sides to obtain an equation of the form

$$\text{a number} \times \text{the unknown} = \text{a number}.$$

3. Divide both sides by the coefficient of the unknown.

Unit 6: Graphs

Plotting the graph of a formula

Book B, Unit 6, Example 1, page 65; also page 67

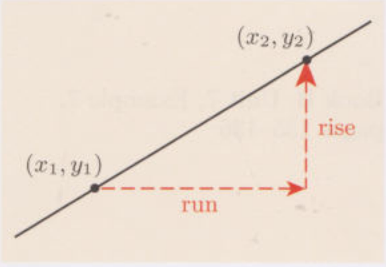
- Construct a table with some values of the independent variable in the first row and corresponding values of the dependent variable in the second row.
- Put the independent variable on the horizontal axis and the dependent variable on the vertical axis, and select a scale that covers all the values in the table.
- Plot the points whose coordinates form the columns of the table and, if appropriate, join them with a straight line or a smooth curve.

Finding the gradient (slope) of a straight line

Book B, Unit 6, page 78

Choose two points (x_1, y_1) and (x_2, y_2) on the line. Then use

$$\text{gradient} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}.$$



The gradient of a vertical line is undefined.
A line that slopes up from left to right has a positive gradient.
A line that slopes down from left to right has a negative gradient.
A horizontal line has gradient 0.

If the axes have units that are different, then the gradient also has units. For example, if the units on the horizontal and vertical axes are seconds and centimetres, respectively, then the units for the gradient are cm/s.

Finding the intercepts and gradient of the line $y = mx + c$

Book B, Unit 6, pages 94–5

- The gradient is the coefficient m.
- The y-intercept is the constant term c.
- The x-intercept is the solution of $mx + c = 0$.

Drawing the line $y = mx + c$ (gradient method)

Book B, Unit 6, page 96

1. Mark the point $(0, c)$ that corresponds to the y-intercept.
2. Count 1 unit right and m units up from the point, and mark the point that you reach. (If m is negative, then you count down rather than up.)
3. Draw the straight line through the two points.

(If the value of m is small, or a fraction, then in step 2 it might be easier to count, say, 2 units right and $2m$ units up, or 3 units right and $3m$ units up, and so on – choose a convenient multiple.)

Drawing the line $y = mx + c$ (two-point method)

Book B, Unit 6, page 98

When you use this strategy you should also find and plot a third point on the line, as a check.

1. Find the coordinates of two points on the line, for example by choosing two values of x and substituting them into the equation to find the corresponding values of y.
2. Plot the points and draw the straight line through them.

Finding the equation of a line

Book B, Unit 6, pages 99–103

- If the line is vertical, then the equation is $x = a$, where a is the x-coordinate of any point on the line.
- If the line is horizontal, then the equation is $y = b$, where b is the y-coordinate of any point on the line.
- In all other cases, do the following.
  1. If the gradient m is not already known, then calculate it by substituting the coordinates of two points (x_1, y_1) and (x_2, y_2) on the line into
  $$m = \frac{y_2 - y_1}{x_2 - x_1}.$$
  Then substitute the gradient into the general equation $y = mx + c$.
  2. If the y-intercept c is not already known, then substitute the coordinates of a point on the line into the equation of the line from step 1, and solve the resulting equation to find c.
  3. Use the values of m and c to write down the equation of the line.

Unit 7: Equations and inequalities

Finding the highest common factor of two or more terms

Book B, Unit 7, Example 7, pages 135–136

1. If the coefficients are integers, write down their highest common factor.
2. For each letter appearing in the terms, write down (if possible) the highest power of the letter that is a factor of all the terms.
3. Write down anything else that is a factor of all the terms.

Taking out a common factor from an expression

Book B, Unit 7, page 137

If most of the terms have a minus sign, then you may wish to take this out as well.

If the common factor that you're taking out is the same as one of the terms, then what's left is 1.

1. Find a common factor of the terms (normally the highest common factor).
2. Write the common factor in front of a pair of brackets.
3. Write what's left of each term inside the brackets.

In complicated cases you may wish to multiply out the brackets again and check that you get the original expression.

Making a variable the subject of an equation

Book B, Unit 7, page 141

Carry out a sequence of steps. In each step, do one of the following:

- do the same thing to both sides
- simplify one side or both sides
- swap the sides.

Aim to do the following, in order.

1. Clear any fractions and multiply out any brackets. To clear fractions, multiply both sides by a suitable expression.
2. Add or subtract terms on both sides to get all terms containing the required subject on one side, and all other terms on the other side.
3. If more than one term contains the required subject, then take it out as a common factor. This gives an equation of the form

   expression × required subject = expression.

4. Divide both sides by the expression that multiplies the required subject.

Remember that you have to do the same thing to the *whole* of each side, namely one of:

- add something
- subtract something
- multiply by something
- divide by something non-zero.

Solving simultaneous equations: graphical method

Book B, Unit 7, page 145

1. Draw the graph of each equation on the same axes, choosing scales so that the intersection point can be seen.
2. The values of the unknowns at the intersection point give the solution.

Determining whether simultaneous equations have a solution

Book B, Unit 7, page 148

Write the equations in the form

$$y = ax + b,$$
$$y = cx + d.$$

- If the constants a and c are *not equal*, then the lines representing the equations are not parallel, so the equations have one solution.
- If the constants a and c are *equal*, then the lines representing the equations are parallel, so the equations do not have a solution. (There is an an exception to this: if the constants b and d are also equal, then the two equations are the same, so there are infinitely many solutions.)

Solving simultaneous equations: substitution method

Book B, Unit 7, page 152

1. Rearrange one of the equations, if necessary, to obtain a formula for one unknown in terms of the other.
2. Use this formula to substitute for this unknown in the other equation.
3. You now have an equation in one unknown. Solve it to find the value of the unknown.
4. Substitute this value into an equation involving both unknowns to find the value of the other unknown.

(Check: Confirm that the two values satisfy the original equations.)

Book B, Unit 7, page 155

Solving simultaneous equations: elimination method

It may be helpful to divide each equation through by any common factor, and clear any fractions, before you start working with the equations.

1. Multiply one or both of the equations by suitable numbers, if necessary, to obtain two equations that can be added or subtracted to eliminate one of the unknowns.
2. Add or subtract the equations to eliminate the unknown.
3. You now have an equation in one unknown. Solve it to find the value of the unknown.
4. Substitute this value into an equation involving both unknowns to find the value of the other unknown.

(Check: Confirm that the two values satisfy the original equations.)

Book B, Unit 7, page 165

Rearranging an inequality

You can do any of the following things to a correct inequality to obtain another correct inequality.

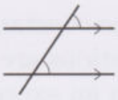
- Do any of the following to *both sides*.
  - Add or subtract a number.
  - Multiply or divide by a *positive* number.
  - Multiply or divide by a *negative* number, *if you reverse the inequality sign*.
- Simplify one side or both sides.
- Swap the sides, *if you reverse the inequality sign*.

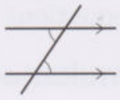
Unit 8: Geometry

Angles (lines)

Book C, Unit 8, page 10
- Angles on a *straight line* add up to 180°. 

Book C, Unit 8, page 13
- *Opposite (X) angles* are equal. 

Book C, Unit 8, page 14
- *Corresponding (F) angles* on parallel lines are equal. 

Book C, Unit 8, page 15
- *Alternate (Z) angles* on parallel lines are equal. 

Angles (polygons)

Book C, Unit 8, page 17
- The interior angles of a triangle add up to 180°.

Book C, Unit 8, page 19
- The angles in an *equilateral triangle* are equal (to 60°).

Book C, Unit 8, page 19
- The base angles in an *isosceles triangle* are equal.

Book C, Unit 8, page 24
- Opposite angles in a *parallelogram* are equal.

Book C, Unit 8, page 24
- A *kite* has a pair of opposite equal angles.

Checking whether two triangles are congruent

Book C, Unit 8, page 34

Check whether the triangles satisfy one of the following conditions.

- The three sides of one triangle are equal to the three sides of the other triangle (SSS).
- Two sides and the included angle of one triangle are equal to two sides and the included angle of the other triangle (SAS).
- Two angles and the included side of one triangle are equal to two angles and the included side of another triangle (ASA).
- Two angles and a side of one triangle in the order angle-angle-side are equal to two angles and a side of the other triangle *in the same order* (AAS).

Checking whether two triangles are similar

Book C, Unit 8, page 42

Check whether the triangles satisfy one of the following conditions.

- Two (and hence three) angles of one triangle are equal to two (and hence three) angles of the other triangle.
- The three sides of one triangle are in proportion to the three sides of the other triangle (that is, their ratios are equal).
- An angle of one triangle is equal to an angle of the other triangle, and the sides containing these angles are in proportion (that is, their ratios are equal).

Finding an unknown side of a triangle using a similar triangle

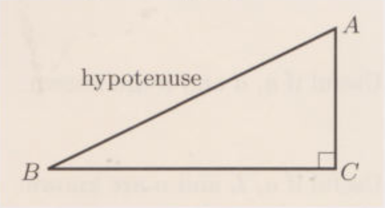
Book C, Unit 8, page 39

1. Show that the triangle is similar to another triangle with known sides.
2. Equate the ratios of corresponding sides.
3. Solve for (the length of) the unknown side.

Pythagoras' Theorem

Book C, Unit 8, page 43

For a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. So, for the right-angled triangle in the margin, $AB^2 = AC^2 + BC^2$.



Converse of Pythagoras' Theorem

Book C, Unit 8, page 46

If a triangle has sides of lengths a, b and c with $a^2 + b^2 = c^2$, then the angle opposite the side of length c is a right angle.

Areas

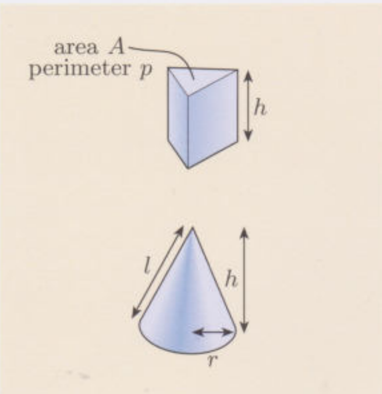
Book C, Unit 8, pages 48–51

- A *rectangle* with sides a and b has area ab.
- A *parallelogram* with base b and perpendicular height h has area bh.
- A *triangle* with base b and perpendicular height h has area $\frac{1}{2}bh$.
- A *trapezium* with parallel sides a and b, and perpendicular height h, has area $\frac{1}{2}(a + b)h$.

Circles

- The circumference of a circle of radius r is $2\pi r$. Book C, Unit 8, page 55
- The area of a circle of radius r is πr^2. Book C, Unit 8, page 56

Book C, Unit 8, page 61



Solids

- A *cuboid* of width w, height h and depth d has volume whd and surface area $2wh + 2wd + 2hd$.
- A *prism* (as shown in the margin) with a cross-section of area A and perimeter p, whose height is h, has volume Ah and surface area $2A + hp$.
- A *cylinder* of radius r and height h has volume $\pi r^2 h$ and surface area $2\pi r^2 + 2\pi rh$.
- A *cone* (as shown in the margin) of radius r, height h and slant height l has volume $\frac{1}{3}\pi r^2 h$ and surface area $\pi r^2 + \pi rl$.
- A *sphere* of radius r has volume $\frac{4}{3}\pi r^3$ and surface area $4\pi r^2$.

Unit 9: Expanding algebra

Sums of sequences

Book C, Unit 9, page 74

- The sum of the first n natural numbers is $\frac{1}{2}n(n + 1)$.

Book C, Unit 9, page 78

- The sum of the first n even numbers is $n(n + 1)$.

Book C, Unit 9, page 73

- The sum of the first n odd numbers is n^2.

Book C, Unit 9, page 76

The nth term of an arithmetic sequence

The nth term of an arithmetic sequence with first term a and common difference d is given by the formula

$$n\text{th term} = a + (n - 1)d.$$

Book C, Unit 9, page 77

The sum of an arithmetic sequence

The sum S of a finite arithmetic sequence with first term a, last term L, common difference d and number of terms n is given by either of the formulas

Useful if a, d and n are known.

$$S = \tfrac{1}{2}n(2a + (n - 1)d)$$

or

Useful if a, L and n are known.

$$S = \tfrac{1}{2}n(a + L),$$

where n, if not already known, can be found from the formula

Useful if a, L and d are known.

$$n = \frac{L - a}{d} + 1.$$

Book C, Unit 9, page 82

$$(a + b)(c + d)$$

1 2 3 4

Multiplying out two brackets

Multiply each term inside the first bracket by each term inside the second bracket, and add the resulting terms. (You can use the acronym FOIL to remember the order: (1) First ac; (2) Outer ad; (3) Inner bc; (4) Last bd.)

Book C, Unit 9, page 84

Square of a bracket

$$(x + p)^2 = x^2 + 2px + p^2 \quad \text{or} \quad (x - p)^2 = x^2 - 2px + p^2$$

Book C, Unit 9, page 86

Difference of two squares

$$(x - p)(x + p) = x^2 - p^2$$

Factorising $x^2 + bx + c$, where b and c are integers

Book C, Unit 9, page 92

1. Decide whether the factorisation is one of the following special types.
   - The quadratic has no constant term, so it factorises like this:
     $$x^2 + bx = x(x + b), \quad \text{for example} \quad x^2 - 6x = x(x - 6).$$
   - The quadratic is a difference of two squares, so it factorises like this:
     $$x^2 - p^2 = (x - p)(x + p), \quad \text{for example} \quad x^2 - 9 = (x - 3)(x + 3).$$
   - The quadratic is a perfect square, so it factorises like this:
     $$x^2 + 2px + p^2 = (x + p)^2, \quad \text{for example} \quad x^2 - 6x + 9 = (x - 3)^2.$$
2. Otherwise, try to fill in the gaps in the brackets on the right-hand side of the equation
   $$x^2 + bx + c = (x\underline{\quad})(x\underline{\quad})$$
   with two integers (positive or negative) whose product is c and whose sum is b. You can search systematically for integers with these properties by:
   - writing down the factor pairs of c, the constant term
   - choosing (if possible) a pair whose sum is b, the coefficient of x.

For some quadratic expressions, the methods given on this page will not lead to a factorisation because a quadratic does not necessarily have a factorisation using integers, even if its coefficients are integers.

You have to find only *one* such pair of integers.

Factorising $ax^2 + bx + c$, where a, b and c are integers

First method

Book C, Unit 9, Example 8, pages 97–98

- For each factor pair r, s of a, try to find a factorisation of the form
  $$ax^2 + bx + c = (rx\underline{\quad})(sx\underline{\quad}),$$
  using a factor pair of c to fill the gaps.
- Multiply out the brackets to check whether the factorisation is successful.
- Keep trying until you obtain a factorisation, or until all factor pairs of c and of a have been exhausted.

Second method

Book C, Unit 9, Example 9, page 98

- Find two numbers p, q whose product is ac and whose sum is b.
- Rewrite the quadratic expression by using the two numbers above to split the term in x:
  $$ax^2 + bx + c = ax^2 + px + qx + c.$$
- Group the four terms in pairs and take out common factors to give the required factorisation:
  $$\begin{aligned} ax^2 + bx + c &= \underbrace{ax^2 + px}\ \underbrace{+\,qx + c} \\ &= \ldots \\ &= \ldots. \end{aligned}$$

For the quadratic
$$2x^2 - x - 6,$$
we need numbers p, q whose product is -12 and whose sum is -1, so we take
$$p = 3, \quad q = -4.$$
Then
$$\begin{aligned} &2x^2 - x - 6 \\ &= \underbrace{2x^2 + 3x}\ \underbrace{-\,4x - 6} \\ &= x(2x + 3) - 2(2x + 3) \\ &= (x - 2)(2x + 3). \end{aligned}$$

Numbers whose product is zero

Book C, Unit 9, page 95

If the product of two or more numbers is 0, then at least one of the numbers must be 0.

Solving a quadratic equation by factorisation

Book C, Unit 9, page 96, 99

For example, the equation

$$2x^2 - x - 6 = 0$$

is equivalent to

$$(x-2)(2x+3) = 0,$$

which gives

$$x - 2 = 0 \quad \text{or} \quad 2x + 3 = 0;$$

that is,

$$x = 2 \quad \text{or} \quad x = -\tfrac{3}{2}.$$

1. Rearrange the equation into the form $ax^2 + bx + c = 0$, if it is not already in this form.
2. Factorise the LHS.
3. Use the fact that if the product of two numbers is zero then at least one of the numbers must be zero.
4. Solve the resulting linear equations.

Simplifying an algebraic fraction

Book C, Unit 9, page 104

Factorise the numerator and denominator (if necessary) and cancel any common factors. For example, $\dfrac{2x^2+6x}{x^2-9} = \dfrac{2x\cancel{(x+3)}^{1}}{(x-3)\cancel{(x+3)}_{1}} = \dfrac{2x}{x-3}$.

Adding or subtracting algebraic fractions

Book C, Unit 9, page 105

1. Make sure that the fractions have a common denominator – if necessary, rewrite each fraction as an equivalent fraction.
2. Add or subtract the numerators.
3. Simplify the answer by cancelling if possible.

Multiplying or dividing algebraic fractions

Book C, Unit 9, page 107

- To multiply two algebraic fractions, multiply the numerators together and multiply the denominators together:
$$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}.$$
- To divide one algebraic fraction by another, multiply the first fraction by the reciprocal of the second fraction:
$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}.$$

In each case, you should cancel any common factors that appear.

Clearing algebraic fractions from an equation

Book C, Unit 9, page 110

Multiply both sides of the equation by a common multiple (often the product) of the denominators of all the fractions that you want to clear.

Book D, Unit 14, page 193

If the equation is of the form $\frac{A}{B} = \frac{C}{D}$, where A, B, C and D are expressions, then you can cross-multiply to obtain $AD = BC$.

Both methods are valid only when the values of the variables are such that the denominators of the fractions are non-zero.

Rearranging an equation when the required subject is raised to a power

Book C, Unit 9, page 116

1. Try to rearrange the equation into the form
$$\left(\text{the required subject}\right)^{\text{a power}} = \text{an expression}.$$
2. Obtain the required subject on its own on the left-hand side by raising both sides of the equation to the reciprocal of the power.

For example, $a^2 = bc$ becomes $a = (bc)^{1/2}$, that is, $a = \sqrt{bc}$.

Unit 10: Quadratics

The graph of the equation $y = ax^2 + bx + c$

Book C, Unit 10, page 143, 147

If a is positive, then the graph is a u-shaped parabola.

If a is negative, then the graph is an n-shaped parabola.

The graph has the same shape as the graph of $y = ax^2$, but shifted.

The graph crosses the y-axis at $(0, c)$.

The axis of symmetry of the graph is $x = -\dfrac{b}{2a}$.

Finding the intercepts of the parabola $y = ax^2 + bx + c$

Book C, Unit 10, page 144

- The x-intercepts are the solutions of the quadratic equation $ax^2 + bx + c = 0$.
- The y-intercept is c.

Finding the vertex of the parabola $y = ax^2 + bx + c$

Book C, Unit 10, page 145, 172

Use any of the following methods.

- Use the formula $x = -b/(2a)$ to find the x-coordinate of the vertex, then substitute into the equation of the parabola to find the y-coordinate.
- Find the x-intercepts; then the value halfway between them is the x-coordinate of the vertex. Find the y-coordinate by substituting into the equation of the parabola.
- Find the completed square form $y = a(x - h)^2 + k$; then the vertex is (h, k).

You can check an answer for the vertex of a parabola by plotting the parabola using Graphplotter and reading off the approximate coordinates of the vertex.

Sketching the graph of a quadratic function

Book C, Unit 10, page 146

1. Decide whether the parabola is u-shaped or n-shaped.
2. Find its intercepts, axis of symmetry and vertex.
3. Plot the features found, and hence sketch the parabola.
4. Label the parabola with its equation, and make sure that the values of the intercepts and the coordinates of the vertex are indicated.

Solving a quadratic equation graphically

Book C, Unit 10, pages 148–149

1. Obtain a graph of the corresponding quadratic function, using a scale on the x-axis that enables you to read x-coordinates to the desired accuracy of the solution.
2. Read off the values of x when $y = 0$, that is, the x-intercepts.

For example, you can use Graphplotter to obtain a graph of a quadratic function.

The quadratic formula

Book C, Unit 10, page 152

The solutions of the quadratic equation

$$ax^2 + bx + c = 0$$

are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

If the coefficients have any common factors, then you may wish to divide the equation through by them before using the formula; and if any of the coefficients are fractions, then you may wish to multiply through by a suitable number to clear them.

Book C, Unit 10, page 157

The number of solutions of a quadratic equation

The quadratic equation $ax^2 + bx + c = 0$ has

- two solutions if $b^2 - 4ac > 0$ (the discriminant is positive)
- one solution if $b^2 - 4ac = 0$ (the discriminant is zero)
- no solutions if $b^2 - 4ac < 0$ (the discriminant is negative).

Completing the square in a quadratic expression

Book C, Unit 10, page 164

- If the quadratic is of the form $x^2 + bx$, write

  $$x^2 + bx = (x+p)^2 - p^2 \quad \text{(where } p \text{ has the value } b/2\text{)}$$

  and evaluate the constant term $-p^2$.

Book C, Unit 10, page 167

- If the quadratic is of the form $x^2 + bx + c$, write

  $$x^2 + bx + c = (x+p)^2 - p^2 + c \quad \text{(where } p \text{ has the value } b/2\text{)}$$

  and collect the constant terms.

Book C, Unit 10, page 171

- If the quadratic is of the form $ax^2 + bx + c$, write

  $$\begin{aligned} ax^2 + bx + c &= a(x^2 + qx) + c \quad \text{(where } q \text{ has the value } b/a\text{)} \\ &= a((x+p)^2 - p^2) + c \quad \text{(where } p \text{ has the value } q/2\text{).} \end{aligned}$$

  Then multiply out the *outer* brackets, and collect the constant terms.

Book C, Unit 10, Example 9, pages 167–168

$4x^2 + 8x - 1 = 0$

$x^2 + 2x - \frac{1}{4} = 0$

$(x+1)^2 - \frac{5}{4} = 0$

$(x+1)^2 = \frac{5}{4}$

$x + 1 = \pm\frac{1}{2}\sqrt{5}$

$x = -1 \pm \frac{1}{2}\sqrt{5}$

Solving a quadratic equation by completing the square

The working for $4x^2 + 8x - 1 = 0$ is in the margin.

1. Divide through by the coefficient of x^2.
2. Complete the square.
3. Get the constant term on the RHS.
4. Take the square root of both sides.
5. Get x by itself on the LHS.

Book C, Unit 10, page 179

Solving a maximisation (or minimisation) problem

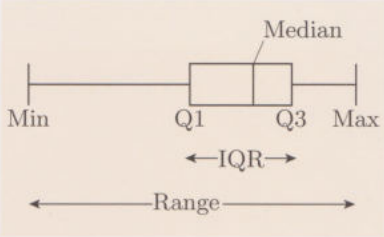
1. Identify the quantity to be maximised (or minimised) and the quantity that it depends on, and denote each quantity by a variable.
2. Find a formula for the variable to be maximised (or minimised) in terms of the variable that it depends on.
3. If this gives a quadratic function, then find the vertex of its graph.

The required maximum (or minimum) value is equal to the second coordinate of the vertex. This value is achieved when the independent variable is equal to the first coordinate of the vertex.

Unit 11: Statistical pictures

Characteristics of boxplots

Book D, Unit 11, page 12, 16



- A boxplot is composed of four sections (two whiskers at either end and two sections within the central box), each of which contains approximately the same number of data values.
- Where a particular boxplot section is narrow, this indicates a dense concentration of the data, whereas a wide section indicates where the data are more sparsely spread.

Uniform random numbers

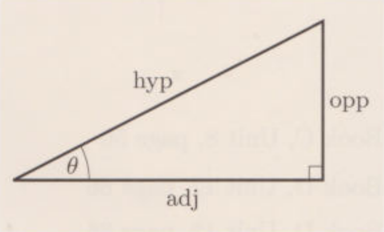
Book D, Unit 11, page 33

- When a fairly small run of uniform random numbers is chosen, the degree of disorderliness in the numbers is often surprisingly high.
- With larger runs, the frequencies tend to settle down and become approximately equal.
- Knowing the extent of random fluctuations for a given sample size provides a benchmark against which to interpret experimental data.

Unit 12: Trigonometry

Trigonometric ratios

Book D, Unit 12, page 61



In a right-angled triangle with an acute angle θ, the sine, cosine and tangent of θ are given by

$$\sin\theta = \frac{\text{opp}}{\text{hyp}}, \quad \cos\theta = \frac{\text{adj}}{\text{hyp}}, \quad \tan\theta = \frac{\text{opp}}{\text{adj}}, \qquad \text{(mnemonic: SOH CAH TOA)}$$

where hyp, opp and adj are the lengths of the hypotenuse, the side opposite θ and the side adjacent to θ, respectively.

Trigonometric identities

Book D, Unit 12, page 74

$$\cos\theta = \sin(90^\circ - \theta) \quad \text{or} \quad \cos\theta = \sin(\tfrac{1}{2}\pi - \theta)$$
$$\sin\theta = \cos(90^\circ - \theta) \quad \text{or} \quad \sin\theta = \cos(\tfrac{1}{2}\pi - \theta)$$

Book D, Unit 12, page 74, 91

$$\tan\theta = \frac{\sin\theta}{\cos\theta}, \quad \text{provided that } \cos\theta \neq 0$$

Book D, Unit 12, page 74

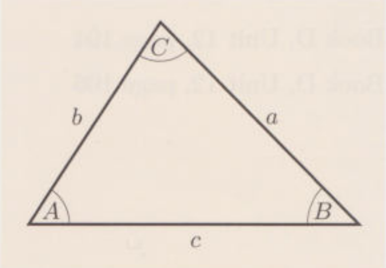
$$\sin^2\theta + \cos^2\theta = 1$$

Book D, Unit 12, page 96

$$\cos(-\theta) = \cos\theta, \qquad \sin(-\theta) = -\sin\theta, \qquad \tan(-\theta) = -\tan\theta$$

Sine Rule

Book D, Unit 12, page 77



In a triangle with sides of length a, b, c and opposite angles A, B, C, respectively:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \qquad \text{or} \qquad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}.$$

Cosine Rule

Book D, Unit 12, page 80

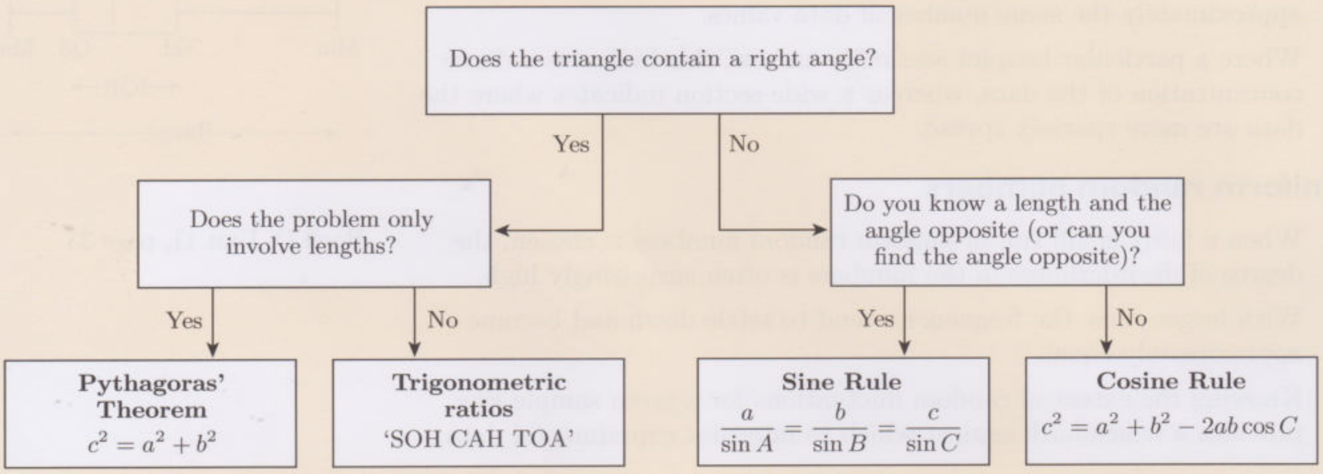
In a triangle with sides of length a, b, c and opposite angles A, B, C, respectively:

$$a^2 = b^2 + c^2 - 2bc\cos A,$$
$$b^2 = c^2 + a^2 - 2ca\cos B,$$
$$c^2 = a^2 + b^2 - 2ab\cos C.$$

Solving a triangle

Book D, Unit 12, page 75, 83

1. Sketch a diagram showing the known measurements.
2. Using the following decision tree, write down an equation relating the unknown to (some of) the known measurements.



3. Solve the equation to find the unknown. If the unknown is an angle then it may be necessary to apply $\sin^{-1}$, $\cos^{-1}$ or $\tan^{-1}$, and in the case of $\sin^{-1}$ you should consider whether the required angle is the obtuse angle $180° - \theta$ rather than the acute angle θ returned by your calculator.

Finding the area of a triangle

- Book C, Unit 8, page 50 If the base b and perpendicular height h are known, calculate $\frac{1}{2}bh$.
- Book D, Unit 12, page 86 If two sides a, b and the included angle θ are known, calculate $\frac{1}{2}ab\sin\theta$.
- Book D, Unit 12, page 86 If the three sides a, b and c are known, use the semi-perimeter $s = \frac{1}{2}(a+b+c)$ to calculate $\sqrt{s(s-a)(s-b)(s-c)}$ (Heron's formula).

Converting between degrees and radians

Book D, Unit 12, page 103

$360° = 2\pi$ radians, so

- angle in radians $= \dfrac{\pi}{180} \times$ angle in degrees,
- angle in degrees $= \dfrac{180}{\pi} \times$ angle in radians.

Finding the area of a sector or the length of an arc

Determine the radius r of the circle with which the arc or sector is associated. Then

- Book D, Unit 12, page 104 arc length $= r\theta$,
- Book D, Unit 12, page 106 area of sector $= \frac{1}{2}r^2\theta$,

where θ (measured in radians) is the angle subtended by the arc or the angle of the sector.

Special angles table

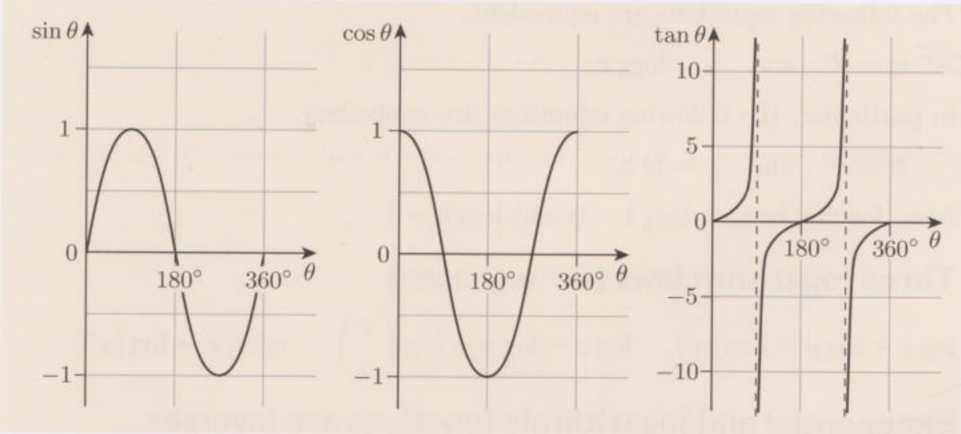
Book D, Unit 12, Subsection 1.4 and page 108

θ in degrees	θ in radians	$\sin\theta$	$\cos\theta$	$\tan\theta$
$0°$	0	0	1	0
$30°$	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
$45°$	$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
$60°$	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$90°$	$\frac{\pi}{2}$	1	0	$-$

Trigonometric graphs

Book D, Unit 12, pages 94–95

The graphs of the sine, cosine and tangent functions are shown below.



Unit 13: Exponentials

Discrete exponential change

Book D, Unit 13, pages 119–120

Suppose that a positive quantity changes in steps, where its value at each step is obtained by multiplying its value at the previous step by the same scale factor b. If the starting number is a, then the value after n steps is ab^n. If $b > 1$ then the quantity grows; if $0 < b < 1$ then it decays.

Scale factors for percentage increases and decreases

Book D, Unit 13, page 121

To increase a number by $r\%$, multiply it by the scale factor $\dfrac{100+r}{100}$.

To decrease a number by $r\%$, multiply it by the scale factor $\dfrac{100-r}{100}$.

Discrete exponential change over different numbers of steps

Book D, Unit 13, page 134

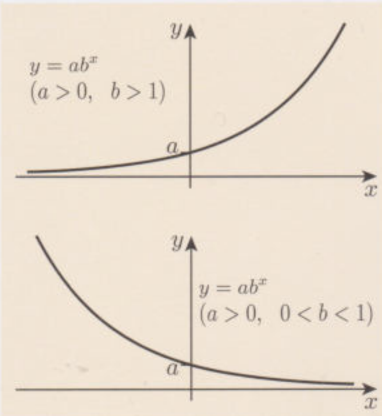
Suppose that a quantity changes by the scale factor b at each step.
Then every i steps, it changes by the scale factor b^i.

Continuous exponential change over different periods of time

Book D, Unit 13, page 137

Suppose that a quantity is subject to continuous exponential change by the scale factor b during each unit of time.
Then over i units of time, it changes by the scale factor b^i.

Book D, Unit 13, page 145



Graphs of equations of the form $y = ab^x$

- If $a > 0$ then the graph lies entirely above the x-axis.

  If also

  $b > 1$, then the graph is an exponential growth curve;
  $0 < b < 1$, then the graph is an exponential decay curve;
  $b = 1$, then the graph is a horizontal line.
- If $a < 0$ then the graph lies entirely below the x-axis and is neither an exponential growth curve nor an exponential decay curve.
- The x-axis is an asymptote (except when $a = 0$ or $b = 1$).
- The y-intercept is a.
- The closer the value of b is to 1 (and the closer the value of a is to 0) the flatter is the graph.

Book D, Unit 13, pages 154–156

Logarithms

The following equations are equivalent.

$$x = b^y \quad \text{and} \quad y = \log_b x.$$

In particular, the following equations are equivalent.

$$x = e^y \quad \text{and} \quad y = \ln x.$$

Also, for any base b, $\log_b 1 = 0$ and $\log_b b = 1$.

Book D, Unit 13, page 161

Here n can be any number, but x and y must be positive since only positive numbers have logarithms.

Three logarithm laws (for any base)

$$\log x + \log y = \log(xy), \quad \log x - \log y = \log\left(\frac{x}{y}\right), \quad n \log x = \log(x^n)$$

Book D, Unit 13, page 165

Exponential and logarithmic functions are inverses

$$\ln(e^x) = x, \quad e^{\ln x} = x$$

More generally,

$$\log_b(b^x) = x, \quad b^{\log_b x} = x$$

Book D, Unit 13, Examples 10 and 11, pages 162–163

Solving an exponential equation

1. Rearrange the equation so that it has the form

   $$p^{\text{an expression involving } x} = q,$$

   where x is the unknown and p and q are numbers.
2. Take logarithms of both sides and use the third logarithm law (above) to write the equation in the form

   $$\text{an expression involving } x \times \log p = \log q.$$
3. Solve the resulting equation for x.

It is usually best to use either logarithms to base 10 or natural logarithms, as these are easily available from your calculator. If $p = e$, use natural logarithms and simplify or evaluate the resulting expression for x.

Book D, Unit 13, page 168

Finding the doubling or halving time of a quantity

Calculate (if not already known) the scale factor b by which the quantity changes during one unit of time.

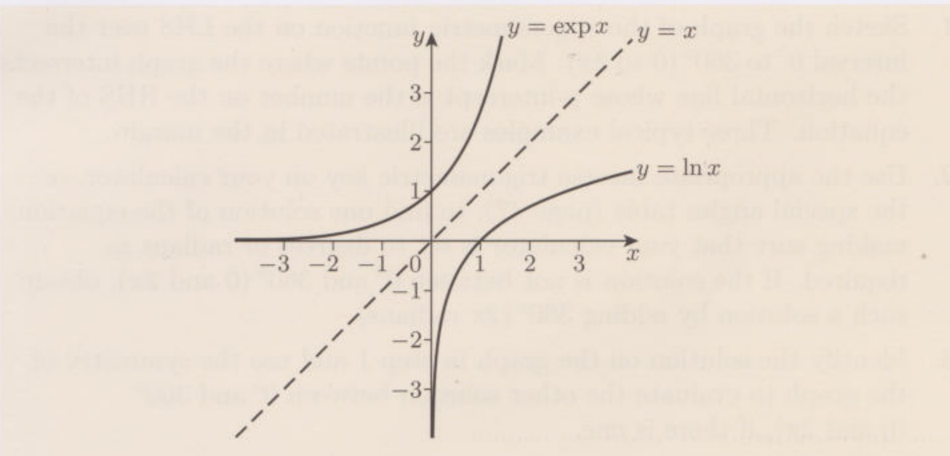
- If the quantity grows (that is, if $b > 1$), then solve $b^i = 2$.
- If the quantity decays (that is, if $0 < b < 1$), then solve $b^i = \frac{1}{2}$.

The solution i is the required doubling or halving time, in the same units of time.

Graphs of inverse functions

Book D, Unit 13, page 170

The graphs of a pair of inverse functions are reflections of each other in the line $y = x$.



Unit 14: Mathematics everywhere

Estimating the probability of an event

Book D, Unit 14, page 181

1. Consider a large number of cases that *could* result in the event that you are interested in.
2. Count the number of cases that *do* result in the event.
3. Divide the number of cases that result in the event by the total number of cases. The quotient provides an estimate of the probability.

Solving trigonometric equations (aided by CAST)

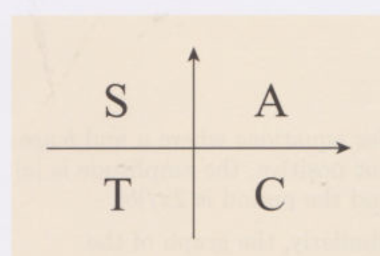
Book D, Unit 14, pages 199–200

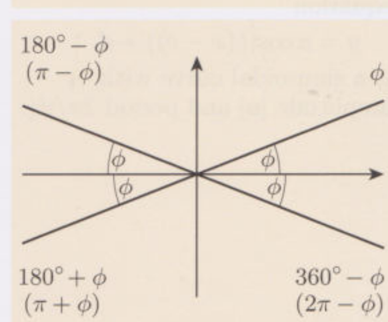
To solve an equation of the form

$$\cos\theta = \text{a number}, \quad \sin\theta = \text{a number} \quad \text{or} \quad \tan\theta = \text{a number},$$

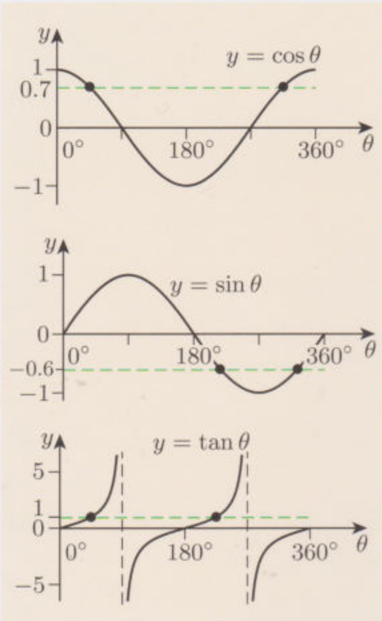
where the number on the RHS is not zero.

1. Use the CAST diagram to find the quadrants of the solutions.
2. Use your calculator, or the special angles table (page 47), to find an angle in the first quadrant whose cosine, sine or tangent (as appropriate) is the magnitude of the number on the RHS. Make sure that your calculator is set to degrees or radians as required.
3. Use the results of steps 1 and 2 and the related angles diagram (expressed in radians if required) to find the solutions in the interval $0°$ to $360°$ (0 to 2π).
4. For further solutions, add integer multiples of $360°$ (2π radians) to the solutions mentioned in step 3.





Book D, Unit 14, pages 206–207



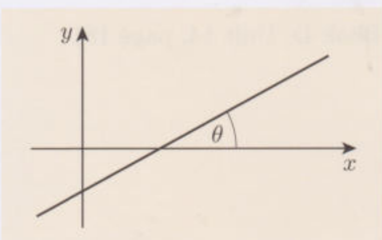
Solving trigonometric equations (aided by sketch graphs)

To solve an equation of the form

$$\cos\theta = \text{a number}, \quad \sin\theta = \text{a number} \quad \text{or} \quad \tan\theta = \text{a number}.$$

1. Sketch the graph of the trigonometric function on the LHS over the interval $0°$ to $360°$ (0 to 2π). Mark the points where the graph intersects the horizontal line whose y-intercept is the number on the RHS of the equation. Three typical examples are illustrated in the margin.
2. Use the appropriate inverse trigonometric key on your calculator, or the special angles table (page 47), to find one solution of the equation, making sure that your calculator is set to degrees or radians as required. If the solution is not between $0°$ and $360°$ (0 and 2π), obtain such a solution by adding $360°$ (2π radians).
3. Identify the solution on the graph in step 1 and use the symmetry of the graph to evaluate the other solution between $0°$ and $360°$ (0 and 2π), if there is one.
4. For further solutions, add integer multiples of $360°$ (2π radians) to the solutions mentioned in step 3.

Book D, Unit 14, page 202



Gradient and angle of inclination of a straight line

For any straight line with angle of inclination θ,

$$\text{gradient} = \tan\theta.$$

(The angle of inclination is measured when the line is drawn on axes with equal scales.)

Book D, Unit 14, pages 225–226

The graph of a general sine function

The graph of the equation

$$y = a\sin(b(x - c)) + d,$$

where a and b are positive, and c and d can take any value, has the following features.

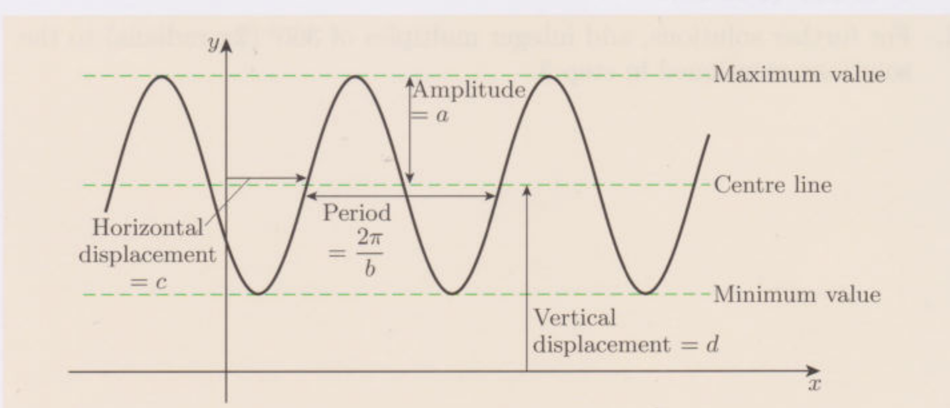
- a is the **amplitude**: the distance between the centre line and the maximum (or minimum) value.
- b tells you the *period*, which is equal to $2\pi/b$.
- c is the **horizontal displacement**: the amount that the graph of $y = a\sin(bx) + d$ is shifted to the right to obtain the graph of $y = a\sin(b(x - c)) + d$. (The shift is to the left if c is negative.)
- d is the **vertical displacement**: the amount that the centre line is shifted up from the x-axis. (The shift is down if d is negative.)

For equations where a and b are not positive, the amplitude is $|a|$ and the period is $2\pi/|b|$.

Similarly, the graph of the equation

$$y = a\cos(b(x - c)) + d$$

is a sinusoidal curve with amplitude $|a|$ and period $2\pi/|b|$.



5 SI units

The Système Internationale d'Unités (SI units) is an internationally agreed set of units and symbols for measuring physical quantities.

Some of these are base units, such as:

metre	symbol m	(measurement of length)
second	symbol s	(measurement of time)
kilogram	symbol kg	(measurement of mass)
kelvin	symbol K	(measurement of temperature).

There are also derived units, which are used for quantities whose measurement combines base units in some way. Some of these are

area	m^2	(metres squared or square metres)
volume	m^3	(metres cubed or cubic metres)
speed	m/s	(metres per second)
acceleration	m/s^2	(metres per second per second).

The metric unit litre (l) is equivalent to $0.001\,m^3$ (or $1000\,cm^3$).

Prefixes may be added to units. Commonly used prefixes are:

n	nano	or	10^{-9}	(e.g. nanogram, ng)
μ	micro	or	10^{-6}	(e.g. microsecond, μs)
m	milli	or	10^{-3}	(e.g. millisecond, ms)
c	centi	or	10^{-2}	(e.g. centimetre, cm)
k	kilo	or	10^{3}	(e.g. kilogram, kg)
M	mega	or	10^{6}	(e.g. megagram, Mg).

6 The Greek alphabet

A	α	alpha	N	ν	nu
B	β	beta	Ξ	ξ	xi
Γ	γ	gamma	O	o	omicron
Δ	δ	delta	Π	π	pi
E	ε	epsilon	P	ρ	rho
Z	ζ	zeta	Σ	σ	sigma
H	η	eta	T	τ	tau
Θ	θ	theta	Y	υ	upsilon
I	ι	iota	Φ	ϕ	phi
K	κ	kappa	X	χ	chi
Λ	λ	lambda	Ψ	ψ	psi
M	μ	mu	Ω	ω	omega